Chapter 7.4: Expected Value and Variance

Wednesday, November 4

Warmup

1. You pay three dollars, roll a fair die, and get back n dollars for rolling the number n. Do you want to play this game?
2. You get a dollar! Do you want to play this game?
3. You flip a fair coin: heads = win a million dollars, tails = lose five hundred thousand dollars. Do you want to play this game?
4. What if you can play the game as many times as you like and don't pay/collect until you're done?

Warmup 2: Weird or Normal?

- HHHHHHHHHHHHHHHHHHHH
- HHHHHHHHHHTTTTTTTTTT
- HTHTHTHTHTHTHTHTHTHT
- THHTTTHTHTTHHHHHTHTT
- 10 coins flipped: 8 heads, 2 tails
- 10000 coins flipped: 6530 heads, 3470 tails
- 10000 coins flipped: 5000 heads, 5000 tails

Basic Properties

1. You roll a pair of dice. What is the expected value of the sum?
2. What is the expected value of the product?
3. Roll a red die and a blue die, and subtract the red number from the blue number. What is the expected value of the difference?
4. Some standardized tests have multiple choice questions with 5 options. You get 1 point for a correct answer, -0.25 points for a wrong answer, and 0 points for no answer. If you gess randomly on every question for a 100-question test, what is your expected score?

Indicator Variables

1. You flip 20 coins. What is the expected number of times you will see the sequence HHH?
2. 10 cows, 10 ducks, and 10 pigs stand in a line in random order. What is the expected number of times a cow will be standing directly in front of a pig?
3. Alice goes to the gym 4 days per week. Bob goes to the gym 3 random days per week. What is the expected number of times per week that Alice and Bob will go to the gym on the same day?

Chebyshev's Inequality

1. The Mets and the Royals are playing a 1001-game series. Suppose the Royals have a 51 percent chance of winning any given game and that the outcomes of the games are independent. What is the expected number of games that the Royals will win?
2. What is the variance?
3. Use Chebyshev's inequality to put a lower bound on the probability that the Royals win the series.