# Chapter 7.4: Expected Value and Variance

Wednesday, November 4

## Warmup

1. You pay three dollars, roll a fair die, and get back $n$ dollars for rolling the number $n$. Do you want to play this game?
2. You get a dollar! Do you want to play this game?
3. You flip a fair coin: heads = win a million dollars, tails = lose five hundred thousand dollars. Do you want to play this game?
4. What if you can play the game as many times as you like and don't pay/collect until you're done?

## Warmup 2: Weird or Normal?

- HHHHHHHHHHHHHHHHHHHH
- HHHHHHHHHHTTTTTTTTTT
- HTHTHTHTHTHTHTHTHTHT
- THHTTTHTHTTHHHHHTHTT
- 10 coins flipped: 8 heads, 2 tails
- 10000 coins flipped: 6530 heads, 3470 tails
- 10000 coins flipped: 5000 heads, 5000 tails

## Basic Properties

1. You roll a pair of dice. What is the expected value of the sum?

   The expected value of a single die is 3.5, so $E(D_1 + D_2) = E(D_1) + E(D_2) = 3.5 + 3.5 = 7$.

2. What is the expected value of the product?

   The rolls are independent, so $E(D_1 D_2) = E(D_1)E(D_2) = 3.5^2 = 12.25$.

3. Roll a red die and a blue die, and subtract the red number from the blue number. What is the expected value of the difference?

   $E(R - B) = E(R) - E(B) = 0$.

4. Some standardized tests have multiple choice questions with 5 options. You get 1 point for a correct answer, -0.25 points for a wrong answer, and 0 points for no answer. If you gess randomly on every question for a 100-question test, what is your expected score?

   The expected value of a single guess is $1 \cdot (1/5) + (-0.25)(4/5) = 0$, so your expected score is zero.

## Indicator Variables

1. You flip 20 coins. What is the expected number of times you will see the sequence HHH?

   There are 18 chances to get HHH (1-3, 2-4, ..., 18-20) and a 1/8 chance of getting HHH in any individual spot, so the expected number of times is $18/8 = 9/4$.

2. 10 cows, 10 ducks, and 10 pigs stand in a line in random order. What is the expected number of times a cow will be standing directly in front of a pig?

   For each cow, there is a 1/30 chance that it is in the back of the line, and if not, then there is a 10/29 chance that it is in front of a pig (10 pigs, 29 remaining spots). So the probability for each individual cow is $(29/30)(10/29) = 1/3$. Since there are 10 cows, the total expected value is 10/3.

3. Alice goes to the gym 4 days per week. Bob goes to the gym 3 random days per week. What is the expected number of times per week that Alice and Bob will go to the gym on the same day?

   On each day that Alice goes to the gym there is a 3/7 chance that Bob goes too. Alice goes 4 times, so the expected overlap is 12/7 days per week.

## Chebyshev's Inequality

1. The Mets and the Royals are playing a 1001-game series. Suppose the Royals have a 51 percent chance of winning any given game and that the outcomes of the games are independent. What is the expected number of games that the Royals will win?

   The expected number of wins is $.51 \cdot 1001 = 510.51$.

2. What is the variance?

   This is a Bernoulli variable, so the variance of a single game is $(.51)(.49) = .2499$ and the variance of 1001 games is about 250.15.

3. Use Chebyshev's inequality to put a lower bound on the probability that the Royals win the series.

   $P(R \leq 500) \leq P(|R - 510.51| \geq 10) \leq 251/100$. ...this is greater than 1, so Chebyshev's inequality is totally useless here.