More Midterm Review

Monday, November 2

Number Theory

1. True or false: there exist integers x and y such that $13x + 41y = 7$.
2. True or false: there exist integers x and y such that $15x + 21y = 7$.
3. Prove or give a counterexample: if $d|a$ and $d|b$ then $d|\gcd(a, b)$.
4. Prove that if n is odd then $n^2 \equiv 1 \pmod 8$.
5. What is $11^{122} \bmod 7$?

Cryptography

1. If we encrypt a number with the scheme $p \mapsto p^{11} \pmod{35}$, then what is the decryption exponent?

Induction

1. Given: if p is prime and $p|ab$ then $p|a$ or $p|b$. Prove: if p is prime and $p|a_1 a_2 \cdots a_n$ then $p|a_i$ for some i.
2. Prove that consecutive Fibonacci numbers are relatively prime.

Counting

1. How many ways are there to buy 7 fruit if your have 10 choices of fruit?
2. What if you want to buy at least one apple and exactly one pear?

Probability

1. What is the chance that a random permutation of the string "COMBINATORICS" will be "MANICROBOTICS"?
2. Urn A has 3 red balls and 1 green ball. Urn B has 2 red balls and 3 green balls. You draw a ball from a random urn and win a prize if you correctly guess which urn you drew the ball from. Which color ball would you rather draw?
3. A loaded six-sided die rolls the number n with probability $n/21$ for $n = 1, 2, \ldots, 6$. If you roll such a die three times, what is the probability that the sum of the three rolls will be 6?