Chapters 1.4-1.6: Quantifiers and Rules of Inference

Wednesday, September 2

Key Notes

- The order of the quantifiers matters!
- Nested negation: $\neg(\forall x)(\exists y)(P(x,y)) \equiv (\exists x)(\neg\exists y)(P(x,y)) \equiv (\exists x)(\forall y)(\neg P(x,y))$
- Conclusion C is valid given premises P as long as $P \to C$ is a tautology.

Warmup: Two-Player Game!

Let A = {"a", "b", "c", . . . , "z"}, let W = $[(x,y) \in A^2|$ "xy" is an English word].

Game 1: Player 1 names a letter x and Player 2 names a letter y. If xy is a word then Player 1 wins. Who has a winning strategy?

Game 2: Now if xy is a word then Player 2 wins. Who has a winning strategy?

English to Quantifier

1. Every integer is either odd or even.
2. Either all integers or odd, or all integers are even.
3. The sum of any two even numbers is even.
4. Everybody doesn't like something, but nobody doesn't like Sara Lee. (2 propositions)
5. Everybody loves my baby, but my baby don't love nobody but me. (2 propositions, maybe 3)
6. The previous lyrics are from an old song popularized by Louis Armstrong. Prove: if we take "everybody" literally, then "my baby" and "me" must actually be the same person!
7. Let M = {rock, paper, scissors}, and let $D(x,y)$ stand for "x defeats y." How can you say "No move in rock-paper-scissors is guaranteed to win" in quantifier notation?

Quantifier to English

Write each of the following statements in English and decide whether they are true or false. The domain is each case is the real numbers.

1. $(\forall x)(x^2 > 0)$
2. $(\exists x)(\forall y)(x > y)$
3. $(\forall x)(\exists y)(x > y)$
4. $(\forall x > 0)(\exists y)(0 < y < x)$
5. $(\forall x)(\exists y)(x + y = 10)$
6. $(\exists x)(\forall y)(x \cdot y = y)$

Write the negations of each of the above statements in quantifier notation and in English.

Deductive Reasoning

A traveler comes upon a snake, who says this: "If you move, I will strike. If you do not move, I will strike." Write this as a single compound proposition and show that it is logically equivalent to "I will strike." Which rule of inference applies here?

If Trevor gets stuck in traffic he will be late to work. If he is late to work he will be fired. He will not get stuck in traffic if *and only if* he takes the shortcut.
Which of the following conclusions are logically valid? Prove the ones that are.

1. If Trevor does not take the shortcut then he will be fired.
2. If Trevor takes the shortcut then he will not be fired.
3. If Trevor is not late to work then he took the shortcut.

If we lose the game, we will either cry or go out for ice cream (maybe both). Which conclusions are logically valid? Prove the ones that are.

1. If we do not cry and do not go out for ice cream, then we did not lose the game.
2. If we lose the game and do not go out for ice cream, then we will cry.
3. If we do not lose the game and we go out for ice cream, then we will not cry.