

Chapters 1.1-1.3: Propositional Logic

Monday, August 31

Key Topics

- Forward: $p \rightarrow q$; Inverse: $\neg p \rightarrow \neg q$; Converse: $q \rightarrow p$; Contrapositive: $\neg q \rightarrow \neg p$
- Forward \equiv contrapositive, inverse \equiv converse
- De Morgan's Law 1: $\neg(p \wedge q) \equiv \neg p \vee \neg q$
- De Morgan's Law 2: $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- Definition of equivalence, tautology, contradiction

Warmup

All humans are mortal, for that is the way of life. All mortals like strawberries, for they are very tasty. Supposing that Odin is not mortal, which of the following *must* be true?

1. Odin is human.
2. Odin is not human.
3. Odin likes strawberries.
4. Odin does not like strawberries.
5. All humans like strawberries.
6. All strawberries like humans.

Show via truth tables that the propositions $p \rightarrow q$ and $\neg p \vee q$ are equivalent.

If-Then Statements

Rewrite the following phrases in the form "If P, then Q." State the contrapositive, inverse, and converse. Decide whether the biconditional "P if and only if Q" is implied.

1. In order to run for president, you must be at least 35.
2. Surrender or die!
3. No shirt, no shoes, no service.
4. A watched pot never boils.
5. You can't order alcohol unless you're over 21.
6. The Warriors will win as long as they play well.
7. The Cubs lose whenever I watch them play.
8. There is a restaurant in Chicago that displays the following sign: "No shirt, no shoes, no pets, no bikes, no service." Please explain.

Equivalences

1. State the inverse, converse, and contrapositive of the following proposition: “If a number n is even and it is divisible by 3 then it is divisible by 6.”
2. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent. Come up with an English example that illustrates the logic.
3. Show that $((p \vee q) \wedge \neg p) \rightarrow q$ is a tautology. Come up with an English example that illustrates the logic.

Knights and Knaves

More Knights and Knaves! A recap: Knights always tell the truth; knaves always lie. If you meet two people, A and B, what can you deduce from their statements?

1. A says “We are both knights” and B says “A is lying!”
2. A says “We are both knaves” and B says nothing.
3. A says “I am a knave or B is a knight” and B says nothing.

Some of the islanders are also werewolves. Both knights and knaves can be werewolves. You are looking for a traveling partner and would rather go with a knight than a knave, but most of all you don’t want a werewolf for a partner (for obvious reasons). Say you meet three people and happen to know that exactly one is a werewolf. They tell you the following:

4. A: At least one of us is a knight.
B: At least one of us is a knave.
C: The werewolf is a knight.

Who should you pick as a partner based on their testimony?