Midterm 1 Review

Wednesday, September 23

Key Topics

- Definition of tautology, contradiction, equivalence
- Put English sentences into propositional logic/predicate logic/quantifier notation.
- De Morgan's Laws for propositions, sets, and quantifiers
- Rules of Inference: "If A then B" is a valid rule of inference if and only if $A \to B$ is a tautology.
- Definition of inverse, converse, contrapositive
- Direct proofs, proofs by contraposition, proving if-and-only-if statements
- Venn Diagrams, relation to propositions about sets
- Proving that $A \subset B$ or $A = B$ for two sets A and B
- Definition of 1-1 and onto functions, proof that a function is/isn't 1-1/onto
- Definitions of divisibility, congruence. Modular arithmetic.
- Definition of prime, gcd, Euclidean Algorithm, Bezout's Theorem

Problems

1. Prove using truth tables that $\neg(p \wedge q) \wedge p$ is equivalent to $\neg q \wedge p$.
2. Let f be the proposition "We go to the farmer's market," d be "We go to the deli", m be "We have money," and c be "We buy cheese." Write the following using f, d, m, c, and logical connectives:
   (a) If we go to the farmer's market and have money, we will buy cheese.
   (b) We will go to the deli if and only if we do not go to the farmer's market.
   (c) We cannot buy cheese if we do not have money.
3. Assume the three statements in the previous question are all true. Which of the following conclusions must be true?
   (a) We will go to the deli or we will go to the farmer's market.
   (b) If we did not buy cheese, then we did not have money.
   (c) If we go to the deli, then we will not buy cheese.
   (d) If we do not go to the deli, then we will buy cheese if and only if we have money.
4. Illustrate with a Venn Diagram, then prove: if $A \subset B$ and $C \subset \overline{B}$ then $A \cap C = \emptyset$.

5. What are the inverse, converse, and contrapositive of the statement "If $a > 0$ and $b > 0$ then $a \cdot b > 0$"? Which of these statements are true and which are false?

6. Prove that $3a + 2$ is even if and only if a is even for integers a.

7. Express in quantifier notation, and prove or disprove:

   (a) For every real number x there is a real number y such that $x + y$ is positive.

   (b) There is a real number x so that $x + y$ is positive for every real number y.

   (c) There is no smallest positive real number a.

8. Evaluate the following:

   (a) $815 \pmod 7$

   (b) $23234 \cdot 101 \pmod 4$

   (c) $(-17) \cdot 82 \pmod 3$

   (d) $5^{88} \pmod 6$

9. Find the greatest common divisor of 184 and 306.

10. Find a function $f : \mathbb{Z} \to \mathbb{Z}$ with each of the following properties:

    (a) f is 1-1 but not onto.

    (b) f is onto but not 1-1.

    (c) f is neither 1-1 nor onto.

    (d) f is 1-1 and onto but not the function $f(n) = n$.