Homework 9 Solutions

Math 55, DIS 101-102

7.1.32 [2 points]

100 people enter a contest with prizes for first, second, and thrid place. What is the probability that Kumar, Janice, and Pedro each win a prize?

The problem can be simplified by ignoring place and selecting only three winners. The probability is then $1/\binom{100}{3}$, or about one in a million.

7.2.16 [0 points]

Show that if E and F are independent then $\overline{E}$ and $\overline{F}$ are also independent.

$$\begin{aligned} p(\overline{E} \cap \overline{F}) &= 1 - p(E \cup F) \\ &= 1 - p(E) - p(F) + p(E \cap F) \\ &= 1 - p(E) - p(F) + p(E)p(F) \\ &= (1 - p(E))(1 - p(F)) \\ &= p(\overline{E})p(\overline{F}). \end{aligned}$$

7.2.28 [2 points]

Probability of a boy is .51, sexes of children are independent. What is the probability in a family of five of...

1. exactly three boys?

$$\binom{5}{3}(.51)^3(.49)^2 \approx .3185$$

2. At least one boy?

$$1 - .49^5 \approx .9718$$

3. At least one girl?

$$1 - .51^5 \approx .9655$$

4. All children of the same sex?

$$.49^5 + .51^5 \approx .06275$$

7.3.4 [2 points]

The probability of selecting an orange ball from box 1 is 3/7 and the probability of selecting an orange ball from box 2 is 5/11. The probability of having selected box 2 given an orange ball is therefore

$$\frac{(5/11)(1/2)}{(5/11)(1/2) + (3/7)(1/2)} = \frac{35}{68}$$

The computation can be simplified using odds notation as well: $(5/11)(1/2) : (3/7)(1/2) = 5/11 : 3/7 = 35 : 33$, so the answer is 35/38.

7.3.12 [2 points]

Probability of sending a 1 is 1/3, with probability of being received correctly 0.8. Probability of sending a zero is 2/3, with probability of being received correctly 0.9.

1. Find the probability that a 0 is received.

   A recieved zero is either a correct zero or an incorrect 1, so by the Sum Rule the probability is $(1/3)(0.2) + (2/3)(0.9) = 2/3$.

2. Find the probability that a 0 was transmitted given that a 0 was received.

   By Bayes' Theorem the probability is $(2/3)(0.9)/(2/3) = 0.9$.