Homework 7 Solutions

Math 55, DIS 101-102

5.3.14 [2 points] *Show that* $f_{n+1}f_{n-1} - f_n^2 = (-1)^n$ *when* n *is a positive integer.*

Base case for proof by induction: When $n = 1$, $f_3 f_0 - f_1^2 = 2 \cdot 0 - 1 = (-1) = (-1)^1$.

Then suppose the formula works for n. It follows that

$$\begin{aligned} f_{n+2}f_n - f_{n+1}^2 &= (f_{n+1} + f_n)f_n - (f_n + f_{n-1})f_{n+1} \\ &= f_{n+1}f_n + f_n^2 - f_n f_{n+1} - f_{n+1}f_{n-1} \\ &= f_n^2 - f_{n+1}f_{n-1} \\ &= -(-1)^n \\ &= (-1)^{n+1}. \end{aligned}$$

This completes the proof by induction.

5.3.30 [0 points] *Prove that in a bit string the string "01" occurs at most one more time than the string "10."*

Look at the strings of consecutive 1s in our bit string S. Each one is responsible for both an "01" (at its beginning) and a "10" (at its end), unless it is at the beginning or the end of S. If it is at the beginning then it produces only a single "10" and if it is at the end then it produces only a single "01."

6.1.16 [2 points] *How many strings are there of four lowercase letters that have the letter* x *in them?*

There are 26^4 strings in all and 25^4 that do not have the letter x. Thus there are $26^4 - 25^4 = 66351$ that have the letter x.

6.1.26 [2 points] *How many strings of four decimal digits...*

1. *do not contain the same digit twice?*

   There are 10 ways to pick the first digit, then 9 for the second, 8 for the third, and 7 for the fourth. Thus $10 \cdot 9 \cdot 8 \cdot 7 = 5040$ in all.

2. *end with an even digit?*

   Half–therefore, 5000.

3. *have exactly three 9s?*

   There are 9 ways to pick the non-9 digit and 4 places to put it, so $9 \cdot 4 = 36$ in all.

6.2.33 [2 points]

Make 200,000 bins, each one standing for the number of hairs on one's head. If we place the $N > 800,000$ people in the bins, PP implies that one bin gets at least $\lceil N/200,000 \rceil \geq N/200,000 > 800,000 = 4$ people. Since $\lceil N/200,000 \rceil$ is an integer greater than 4, it must be at least 5.