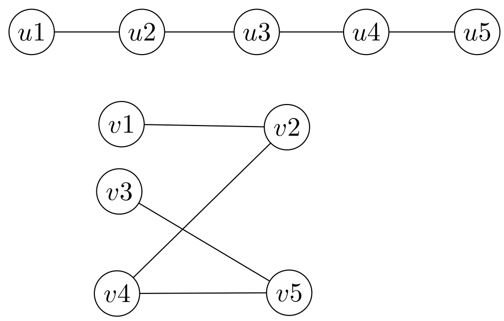
Homework 14 Solutions

Math 55, DIS 101-102

10.3.34 [2 points]

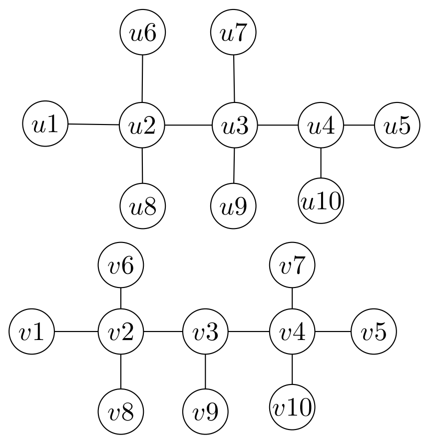
Given an isomorphism or prove that none exists:



There are two possible isomorphisms here: $\{(u1, v1), (u2, v2), (u3, v4), (u4, v5), (u5, v3)\}$ and $\{(u1, v3), (u2, v5), (u3, v4), ($

10.3.42 [2 points]

Give an isomorphism or prove that none exists:



The two graphs are not isomorphic: in the first graph the two vertices of degree 4 are adjacent but in the second graph they are not.

10.4.28 [2 points]

Show that every connected graph with n vertices has at least $n-1$ edges.

Go by induction: when $n = 1$, a graph with one vertex has zero edges.

Then suppose the result holds for n, and consider a graph with $n+1$ vertices. If all vertices have degree 2 or more, then (by the handshake theorem) the graph has at least $2n+2$ edges and we are done. So suppose that there is a vertex of degree 1 (no vertex can have degree zero, since then that vertex would not be connected to the rest of the graph). If we remove this vertex, the remaining graph will still be connected and by inductive assumption will have at least $n-1$ edges. Since we removed one vertex and one edge with it, the original graph had to have at least $(n-1)+1 = n$ edges. This completes the proof by induction.

10.4.38 [2 points]

Show that an edge in a simple graph is a cut edge if and only if this edge is not part of any simple circuit in the graph.

Call the edge (u, v). If the edge is not a cut edge, then there is a simple path from u to v that does not use the edge (u, v). Concatenating this path with the edge (u, v) would make a simple circuit.

Then the converse: if (u, v) is part of a simple circuit, then removing the edge (u, v) turns this simple circuit into a simple path from u to $v \ldots$ call this path P.

But this means that any path $P_1 + (u, v) + P_2$ in the graph can be replaced by the path $P_1 + P + P_2$, and so removing the edge (u, v) will not disconnect any pair of vertices in the graph. Therefore (u, v) is not a cut edge.