Homework 12 Solutions

Math 55, DIS 101-102

8.4.8 [2 points]

For each function, find a closed formula for the sequence it determines:

1. $(x^2+1)^3$: $a_n = \begin{cases} \binom{3}{n/2} & 2|n \\ 0 & 2 \nmid n \end{cases}$
2. $(3x-1)^3$: $a_n = \binom{3}{n}(3)^n(-1)^{3-n}$
3. $1/(1-2x^2)$: $a_n = \begin{cases} 2^{n/2} & 2|n \\ 0 & 2 \nmid n \end{cases}$
4. $x^2/(1-x)^3$: $a_n = \binom{n}{2}$

8.6.9 [2 points] How many ways are there to give six *different* toys to three children so that each child gets at least one toy?

There are 3^6 ways to give out the toys, minus $3 \cdot 2^6$ ways to give out the toys excluding at least one child, plus $3 \cdot 1^6$ ways to give out the toys excluding at least two children, minus 0 ways to exclude all three children, so 540 ways in all.

8.6.22 [2 points] Use inclusion-exclusion to find $\phi(pq)$.

There are pq numbers between 1 and pq minus p that are divisible by q minus q that are divisible by p, plus 1 for pq, so the answer is $pq - p - q + 1 = (p-1)(q-1)$.

9.1.10 [2 points] Give an example of a relation that is

1. both symmetric and antisymmetric: Let $A = \{1, 2, 3\}$. Then $R = \{(1,1)\}$ will do (or even $R = \emptyset$!).
2. Neither symmetric nor antisymmetric: Let $R = \{(1,2), (2,1), (2,3)\}$. It is not symmetric because $(2,3) \in R$ but $(3,2) \notin R$, and it is not antisymmetric because $(1,2), (2,1) \in R$ but $1 \neq 2$.

   Remark: It seems that a lot of people were confusing "antisymmetric" with "not symmetric" for 10b. It may help to think of the relations $\geq$ or $\subseteq$: the only way we can have $a \leq b$ and $a \geq b$ is if $a = b$. Similarly, $A \subseteq B$ and $B \subseteq A$ implies that $A = B$.