Homework 1 Solutions

Math 55, DIS 101-102

(1.2.7) (3 points)

1. $q \to p$
2. $q \wedge \neg p$

(1.2.25) (3 points)

One knight, one knave, one spy. The statements:

A: “I am the knight.”
B: “I am the knave.”
C: “B is the knight.”

Many lost points on the Knights and Knaves problem for simply writing an answer without any explanation. This may seem like a small issue, but it's important for two reasons: first, it was possible for the puzzles to have multiple valid solutions. So if the solution you propose is the unique solution, you should explain why that is the case.

Second, being able to read and write proofs is an important part of this class, especially the first few weeks. For many problems you will find that the process of deduction that leads to a particular answer is more interesting than the answer itself! In this case, it isn't really important who is a knight and who is a knave, but it is critical that you learn to present arguments in a clear and convincing manner.

Okay, enough lecturing. There are lots of ways to find the right answer, but one of the quickest is the following: “B claims to be a knave, but neither a knight nor a knave can make such a claim. Thus B is the spy. C is lying about B and so must be the knave (since B is already the spy). A is therefore the knight.”

A few other remarks:

1. Some people said that A was the knight with the following reasoning: “If A is the knight, then he is telling the truth about himself, which fits with him being a knight.” This means that it is *possible* for A to be a knight, but based on this evidence alone A could just as easily be a lying knave or spy.
2. Some people tried all six possible (knight, knave, spy) combinations and ruled out the impossible ones one by one, leaving only the correct solution. This solution is valid but discouraged. For stylistic reasons, it is generally better to avoid looking at all possible cases unless you have no other choice.

(1.3.8) (3 points)

1. The negation of “Kwame will take a job in industry or go to graduate school” is “Kwame will not take a job in industry and will not go to graduate school,” or “Kwame will neither go to industry nor go to graduate school.”
2. The negation of “Yoshiko knows Java and calculus” is “Yoshiko does not know Java or does not know calculus,” or (more naturally, though ignoring De Morgan's Law) “Yoshiko does not know both Java and calculus.”

   The answer “Yoshiko does not know Java or calculus” is incorrect, as this implies that Yoshiko knows neither. Because of the structure of English sentences, you cannot negate the proposition simply by switching “does” with “does not” and “and” with “or.”
3. The negation of “James is young and strong” is “James is not young or not strong.” The answer “James is not young or strong” is incorrect since this implies that he is neither.

4. The negation of “Rita will move to Oregon or Washington” is “Rita will not move to Oregon and will not move to Washington.” The answer “Rita will not move to Oregon and Washington” is incorrect, since this implies that she will not move to *both* states where we want to say that she will not move to *either*.

(1.3.22) (1 point)

Many people gave an answer using truth tables. This works but using a series of equivalences is preferable since it’s a little closer to an argument in natural English. One solution:

$$\begin{aligned}(p \to q) \land (p \to r) &\equiv (\neg p \lor q) \land (\neg p \lor r)\\ &\equiv \neg p \lor (q \land r)\\ &\equiv p \to (q \land r)\end{aligned}$$