Math 54 Midterm 2
July 30, 2019
50 Minutes

Name:_____

1	
2	
3	
4	
5	
6	
7	
8	

1. Compute the inverse of
$$\begin{pmatrix} 3 & 1 \\ 1 & 2 \end{pmatrix}$$

2. Let A be an 3×3 matrix that is invertible. What is the dimension of the image of A? What is the dimension of the kernel of A?

3. Let $x = (-n, -(n-1), -(n-2), -(n-3), \ldots, n-1, n)$. x is a vector in $\mathbb{R}^{2n+1}$. Let $e_1, \ldots, e_{2n+1}$ be the standard basis vectors of $\mathbb{R}^{2n+1}$. In terms of n and i, what is $x \cdot e_i$?

4. Let

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 6 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

What are the dimensions of the image and kernel of A? What are the eigenvalues of A? Write down an eigenbasis for A.

5. Let

$$A = \begin{pmatrix} -3 & 0 & 3 \\ -2 & 1 & 1 \\ -6 & 0 & 6 \end{pmatrix}$$

What is A^{100}? (You can write the answer in terms of powers like 5^{100}).

6. What is the angle between the two lines

$$\begin{cases} x + y - z = 0 \\ x + 2y - z = 0 \end{cases} \quad \text{and} \quad \begin{cases} x + y = 0 \\ z = 0 \end{cases} \quad ?$$

(There are two right answers, since there are two angles between the lines.)

7. (a) Find an orthonormal basis of the plane $x + y + z = 0$.

(b) What is the orthogonal projection of $(2, 0, 2)$ onto this plane?

8. You write a program to compute the singular value decomposition of a matrix. After inputting a certain 3×2 matrix, it outputs the following:

$$U = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 5 & 0 \\ 0 & 2 \\ 0 & 0 \end{pmatrix}$$

$$V = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Evidently there's a bug in the code. Why is this not a correct singular value decomposition?