Math 54 Practice Midterm 2

1. Solve the following system of linear equations in four variables

$$\begin{cases} 2x_1 - 2x_2 - x_3 - x_4 = -5 \\ x_1 - x_4 = 0 \\ -2x_1 + x_3 + 2x_4 = 0 \\ 4x_1 - 2x_2 - 2x_3 - 4x_4 = -6 \end{cases}$$

2. For each linear system below, determine if each solution set is a point, a line, a plane, $\mathbb{R}^3$, or if there is no solution. You do not need to justify your answer.

   (a)
   $$\begin{cases} x - y = 0 \\ y - z = 0 \\ z - x = 0 \end{cases}$$

   (b)
   $$\begin{cases} y = 0 \\ x + 2y = 1 \end{cases}$$

   (c)
   $$\begin{cases} 7x + 5y - 2z = 0 \\ -4x - 2y + 2z = 0 \\ 3x + 2y + z = 0 \end{cases}$$

   (d)
   $$\begin{cases} 2x - 3y + 5z = 1 \\ 4x - 6y + 10z = 1 \end{cases}$$

3. For each set of vectors, determine if it is linearly independent or linearly dependent:

   (a)
   $$\begin{pmatrix} 1 \\ -5 \\ -4 \\ 8 \end{pmatrix}, \begin{pmatrix} -1 \\ 9 \\ 8 \\ -16 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -1 \\ 2 \end{pmatrix}$$

   (b)
   $$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -4 \\ 0 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -5 \\ -1 \\ 2 \\ 1 \end{pmatrix}$$

(c)

$$\begin{pmatrix} -9 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

4. Suppose $f : \mathbb{R}^4 \to \mathbb{R}^4$ is given by the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

   If f surjective? Injective?

5. Find a bases for the image and the kernel of the linear map from $\mathbb{R}^4$ to $\mathbb{R}^3$ given by the matrix

$$\begin{pmatrix} 2 & -4 & 6 & 4 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 \end{pmatrix}.$$

6. Consider the linear map from $\mathbb{R}^5$ to $\mathbb{R}^4$ given by

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

   Write down a vector in the kernel of this linear map. What geometric shape is the kernel of this map?

7. Write down a basis for the plane in $\mathbb{R}^4$ given by the solution to

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ 3x_2 + 2x_3 + x_4 = 0 \end{cases}$$

8. Let C be the set of points in $\mathbb{R}^3$ with coordinates between 0 and 1 (inclusive). C forms a cube. What sort of shape is the image of C under the map $\mathbb{R}^3 \to \mathbb{R}^2$ given by the matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}?$$