

## Math 54 Practice Midterm 1

1. Solve the following system of linear equations in four variables

$$\begin{cases} x_1 + x_2 + x_4 = 2 \\ x_1 + 4x_2 - x_3 + 2x_4 = 3 \\ -2x_1 - x_2 - 2x_4 = -4 \\ x_3 = -1 \end{cases}$$

2. For each linear system below, determine if each solution set is a point, a line, a plane,  $\mathbb{R}^3$ , or if there is no solution. You do not need to justify your answer.

(a)

$$\begin{cases} x = 2 \\ y + z = 2 \end{cases}$$

(b)

$$\begin{cases} 2x + y + z = 1 \\ x + y + z = -1 \end{cases}$$

(c)

$$\begin{cases} x + 2y + 2z = 1 \\ 4x + 6y + 6z = 0 \\ 8y = -1 \end{cases}$$

(d)

$$\begin{cases} 4x + 7y + 14z = -4 \\ x - 7z = -1 \\ -7x - 8y + z = 7 \end{cases}$$

3. For each set of vectors, determine if it is linearly independent or linearly dependent:

(a)

$$\begin{pmatrix} 2 \\ 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ -3 \\ 6 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -2 \\ 4 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -7 \\ -7 \\ 0 \end{pmatrix}, \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}$$

(d)

$$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}$$

4. Write down two planes in  $\mathbb{R}^4$  that do not intersect.

5. Do the lines

$$t \mapsto t \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

and

$$t \mapsto t \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ -6 \\ 2 \end{pmatrix}$$

intersect? Justify your answer.

6. Write down a basis for the plane in  $\mathbb{R}^3$  given by

$$x + 2y + 3z = 0$$

7. Suppose  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  is defined by

$$f \begin{pmatrix} x \\ y \\ z \end{pmatrix} \rightarrow \begin{pmatrix} x + y + z \\ 0 \end{pmatrix}$$

Is  $f$  surjective? Injective?

8. What is the image of the linear map from  $\mathbb{R}^3$  to  $\mathbb{R}^2$  given by the matrix

$$\begin{pmatrix} 1 & 2 & -1 \\ 3 & 6 & -3 \end{pmatrix}?$$