**Math 54 Midterm 1**

July 9, 2019

50 Minutes

Name:________________________

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |

1. Solve the following system of linear equations in four variables

$$\begin{cases} -2x_1 + 2x_2 - 2x_3 + x_4 = 7 \\ x_2 = 1 \\ -4x_1 - 2x_3 + x_4 = 7 \\ -2x_1 + 2x_2 - x_3 = 5 \end{cases}$$

2. For each linear system below, determine if each solution set is a point, a line, a plane, $\mathbb{R}^3$, or if there is no solution. You do not need to justify your answer.

   (a)
   $$x + y + z = 0$$

   (b)
   $$\begin{cases} x + y + z = 2 \\ 2x + 2y + 2z = 4 \end{cases}$$

   (c)
   $$\begin{cases} x + y + z = 0 \\ x + y = -1 \end{cases}$$

   (d)
   $$\begin{cases} 4x + 9y - 10z = 2 \\ -3x - 6y + 7z = 0 \\ x + 5y + 5z = -1 \end{cases}$$

3. Multiply the matrices
$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

4. (a) Write down a linear map (or a matrix representing that map) from $\mathbb{R}^5$ to $\mathbb{R}^5$ whose image is a plane.
   (b) Write down a linear map (or a matrix representing that map) from $\mathbb{R}^5$ to $\mathbb{R}^5$ whose image is a point.

5. Let

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

write down a vector in $\mathbb{R}^3$ not in the span of $\mathbf{v}_1$ and $\mathbf{v}_2$ and justify your answer.

6. For each set of vectors, determine if it is linearly independent or linearly dependent. Justify your answers.

   (a)
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

   (b)
$$\begin{pmatrix} -2 \\ 2 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \\ -2 \end{pmatrix}$$

7. Consider the following two planes:

$$\begin{pmatrix} t \\ s \end{pmatrix} \mapsto \begin{pmatrix} t+s \\ s+1 \\ t \end{pmatrix}$$

$$\begin{pmatrix} t \\ s \end{pmatrix} \mapsto \begin{pmatrix} 2s \\ s-t-1 \\ t+s \end{pmatrix}$$

(a) Are either of the planes linear subspaces of $\mathbb{R}^3$? Comment briefly.

(b) Do the planes intersect? If so, parametrize the line they intersect in.

8. (a) Suppose $f : \mathbb{R}^2 \to \mathbb{R}^3$ is defined by

$$f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \\ x \end{pmatrix}$$

Is $f$ surjective? Injective?

(b) Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is defined by

$$f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x + y \\ x - y \end{pmatrix}$$

is $f$ surjective? Injective?