Math 54 Homework 8

Due Tuesday August 6

1.

$$f(x) = C_1 e^{2x} + C_2 e^{-x}$$

2. (fixed a mistake 8/12)

$$f(x) = C_1 e^{2x} + C_2 e^{-x}$$
$$0 = f(0) = C_1 + C_2$$
$$1 = f'(0) = 2C_1 - C_2$$
$$\Rightarrow C_1 = \frac{1}{3},\ C_2 = -\frac{1}{3}$$
$$\Rightarrow f(x) = e^{2x} + e^{-x}$$

3.

$$f(x) = C_1 e^{3x} + C_2 e^{x}$$

4.

$$f(x) = C_1 e^{3x} + C_2 e^{x}$$
$$0 = f(0) = C_1 + C_2$$
$$0 = f'(0) = 3C_1 + c_2$$
$$\Rightarrow C_1 = C_2 = 0$$
$$\Rightarrow f(x) = 0$$

5.

$$f(x) = C_1 e^{(3+i)x} + C_2 e^{(3-i)x}$$
$$0 = f(0) = C_1 + C_2$$
$$2 = f'(0) = (3+i)C_1 + (3-i)C_2$$
$$\Rightarrow C_1 = -i,\ C_2 = i$$
$$\Rightarrow f(x) = -ie^{(3+i)x} + ie^{(3-i)x}$$

(same thing as $-ie^{3x}(e^{ix} - e^{-ix}) = 2e^{3x}\sin(x)$.)

6. Homogeneous solution:

$$f_0(x) = C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x}$$

For a particular solution, guess $f_p(x) = Ae^x$. Then

$$\frac{d^2}{dx^2} f_p(x) - 2f_p(x) = e^x$$
$$Ae^x - 2Ae^x = e^x \Rightarrow A = -1$$
$$f(x) = f_p(x) + f_0(x) = -e^x + C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x}$$

7. Homogeneous solution:

$$f_0(x) = C_1 e^{2x} + C_2 e^{x}$$

For a particular solution, guess $f_p(x) = A\cos(x) + B\sin(x)$

$$\frac{d^2 f_p}{dx^2}(x) - 3\frac{df_p}{dx}(x) + 2f_p(x) = \cos(x)$$

$$-A\cos(x) - B\sin(x) - 3(-A\sin(x) + B\cos(x)) + 2(A\cos(x) + B\sin(x)) = \cos(x)$$

$$\Rightarrow \begin{cases} A - 3B = 1 \\ 3A + B = 0 \end{cases}$$

$$\Rightarrow A = \frac{1}{10},\ B = -\frac{3}{10}$$

$$\Rightarrow f(x) = f_p(x) + f_0(x) = \frac{1}{10}\cos(x) - \frac{3}{10}\sin(x) + C_1 e^{2x} + C_2 e^{x}$$

8. Homogeneous solution:

$$f_0 = C_1 e^{2x} + C_2 e^{-2x}$$

For a particular solution, guess $f_p(x) = Ax^2 + Bx + C$

$$2A - 4(Ax^2 + Bx + C) = x^2 + x + 1$$

$$\Rightarrow \begin{cases} -4A = 1 \\ -4B = 1 \\ 2A - 4C = 1 \end{cases}$$

$$\Rightarrow A = -\frac{1}{4},\ B = -\frac{1}{4},\ C = -\frac{3}{8}$$

$$\Rightarrow f(x) = f_p(x) + f_0(x) = -\frac{1}{4}x^2 - \frac{1}{4}x - \frac{3}{8} + C_1 e^{2x} + C_2 e^{-2x}$$

9. Homogeneous solution:

$$f_0(x) = C_1 e^{(3+i)x} + C_2 e^{(3-i)x}$$

For a particular solution, guess $f_p(x) = Ax + B$

$$0 - 6A + 10(Ax + B) = 5x$$

$$\Rightarrow A = \frac{1}{2},\ B = \frac{3}{10}$$

$$\Rightarrow f(x) = f_p(x) + f_0(x) = \frac{1}{2}x + \frac{3}{10} + C_1 e^{(3+i)x} + C_2 e^{(3-i)x}$$

10. From the last problem,

$$f(x) = \frac{1}{2}x + \frac{3}{10} + C_1 e^{(3+i)x} + C_2 e^{(3-i)x}$$

Need to solve for C_1 and C_2:

$$\frac{3}{10} = f(0) = 0 + \frac{3}{10} + C_1 + C_2$$

$$\frac{3}{2} = f'(0) = \frac{1}{2} + (3+i)C_1 + (3-i)C_2$$

$$\Rightarrow C_1 = -\frac{i}{2},\ C_2 = \frac{i}{2}$$

$$\Rightarrow f(x) = f_0(x) + f_p(x) = \frac{1}{2}x + \frac{3}{10} - \frac{i}{2}e^{(3+i)x} + \frac{i}{2}e^{(3-i)x}$$

11. (fixed a mistake 8/11) Homogeneous solution:

$$f_0(x) = C_1 + C_2 e^{-x}$$

For a particular solution, guess $f_p(x) = A\sin(3x) + B\cos(3x)$

$$-9A\sin(3x) - 9B\cos(3x) + 3A\cos(3x) - 3B\sin(3x) = \sin(3x)$$

$$\Rightarrow \begin{cases} -9A - 3B = 1 \\ 3A - 9B = 0 \end{cases}$$

$$\Rightarrow A = -\frac{1}{10},\ B = -\frac{1}{30}$$

$$\Rightarrow f(x) = f_p(x) + f_0(x) = -\frac{1}{10}\sin(3x) - \frac{1}{30}\cos(3x) + C_1 + C_2 e^{-x}$$

12. (fixed a mistake 8/12) Homogeneous:

$$C_1 e^{(2+\sqrt{3})x} + C_2 e^{(2-\sqrt{3})x}$$

Particular:

$$Ax^2 + Bx + C$$

and

$$f(x) = -\frac{1}{2}x^2 + x - \frac{3}{2} + \left(-\frac{\sqrt{3}}{3} - \frac{1}{4}\right)e^{(2+\sqrt{3})x} + \left(\frac{3}{4} + \frac{\sqrt{3}}{3}\right)e^{(2-\sqrt{3})x}$$