Math 54 Homework 5 Solutions

5.1-1 The eigenvalues λ satisfy

$$\det\begin{pmatrix}3-\lambda & 2\\ 3 & 8-\lambda\end{pmatrix}=0$$

and

$$\det\begin{pmatrix}1 & 2\\ 3 & 6\end{pmatrix}=0$$

so 2 is an eigenvalue.

5.1-6

$$\begin{pmatrix}3 & 6 & 7\\ 3 & 3 & 7\\ 5 & 6 & 5\end{pmatrix}\begin{pmatrix}1\\ -2\\ 1\end{pmatrix}=\begin{pmatrix}-2\\ 4\\ -2\end{pmatrix}=-2\begin{pmatrix}1\\ -2\\ 1\end{pmatrix}$$

so $(1,-2,1)$ is an eigenvector with eigenvalue -2.

5.1-9 A vector in the $\lambda=5$ eigenspace is an element of the kernel of

$$\begin{pmatrix}5-5 & 0\\ 2 & 1-5\end{pmatrix}=\begin{pmatrix}0 & 0\\ 2 & -4\end{pmatrix}$$

one such vector is $(2,1)$. A vector in the $\lambda=1$ eigenspace is an element of the kernel of

$$\begin{pmatrix}5-1 & 0\\ 2 & 1-1\end{pmatrix}.$$

One such vector is $(0,1)$. Since these two eigenspaces are lines, these two vectors form bases for the two eigenspaces.

5.1-13 Again, find vectors in the kernel of

$$\begin{pmatrix}4-\lambda & 0 & 1\\ -2 & 1-\lambda & 0\\ -2 & 0 & 1-\lambda\end{pmatrix}$$

for $\lambda=1,2,3$. When $\lambda=1$, a nonzero vector in the kernel is $(0,1,0)$. When $\lambda=2$, a nonzero vector in the kernel is $(-1,2,2)$. When $\lambda=3$, a nonzero vector in the kernel is $(1,-1,-1)$.

5.1-19 The columns of the matrix are linearly dependent, hence there is a nonzero element in the kernel of the matrix. Hence there is an eigenvector with eigenvalue 0.

5.3-1

$$A^4=PD^4P^{-1}=\begin{pmatrix}5 & 7\\ 2 & 3\end{pmatrix}\begin{pmatrix}16 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}3 & -7\\ -2 & 5\end{pmatrix}=\begin{pmatrix}226 & -525\\ 90 & -209\end{pmatrix}$$

5.3-2

$$\begin{pmatrix} 2 & -3 \\ -3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/16 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 151/16 & 45/8 \\ -225/16 & -67/8 \end{pmatrix}$$

5.3-7 $(1,3)$ and $(0,1)$ form an eigenbasis for eigenvalues 1 and -1, respectively. Therefore

$$\begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$$

5.3-10 $\lambda = 5, -2$ are the two eigenvalues. $(1,1)$ is an eigenvector for $\lambda = 5$. $(-3,4)$ is an eigenvector for $\lambda = -2$. Therefore

$$\begin{pmatrix} 2 & 3 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 4/7 & 3/7 \\ -1/7 & 1/7 \end{pmatrix}$$