Math 54 Homework 4 Solutions

Here are the solutions to both posted versions of the homework.

2.2-7

$$A^{-1} = \frac{1}{2}\begin{pmatrix} 12 & -2 \\ -5 & 1 \end{pmatrix}$$

so if $A\mathbf{x} = \mathbf{b}_1$ then

$$\mathbf{x} = A^{-1}\mathbf{b}_1 = \begin{pmatrix} -9 \\ 4 \end{pmatrix}$$

and if $A\mathbf{x} = \mathbf{b}_2$ then

$$\mathbf{x} = A^{-1}\mathbf{b}_2 = \begin{pmatrix} 11 \\ -5 \end{pmatrix}$$

and if $A\mathbf{x} = \mathbf{b}_3$ then

$$\mathbf{x} = A^{-1}\mathbf{b}_3 = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

and if $A\mathbf{x} = \mathbf{b}_4$ then

$$\mathbf{x} = A^{-1}\mathbf{b}_4 = \begin{pmatrix} 13 \\ -5 \end{pmatrix}$$

2.2-31

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right)$$

so the inverse is

$$\begin{pmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{pmatrix}$$

2.2-32

$$\left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right) \to \left(\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & -2 & 2 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & -7 & 2 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 10 & -2 & 1 \end{array}\right)$$

at this point you see there is no way to row reduce to get the identity matrix on the left, so this matrix is not invertible. Another way to see this is to compute the determinant:

$$\det\begin{pmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{pmatrix} = 0$$

The determinant is 0 if and only if the matrix is not invertible.

Another way to do this is to observe that the columns are not linearly independent (the first and third add up to negative one times the second) so there is a nontrivial element in the kernel of the matrix, so the matrix is not injective and hence not invertible.

2.2-38

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}$$

(remember that Ae_i is the ith column of A). It is not possible that $CA = I$ for some 4×2 matrix C since A is not injective and hence CA is not injective. But I is always injective.

3.1-1 Expand down the first column:

$$3(-3-10) - 2(-20) = 1$$

3.1-3 Expand across bottom row:

$$(-4-3) - 3(4-9) - (2+6) = 0$$

3.1-5 Expand down second column:

$$-3(20-18) - (6+12) = -24$$

3.1-7 Expand down third column:

$$-2(28-27) + 3(20-18) = 4$$

3.1-9 Expand across third row:

$$3\det\begin{pmatrix} 0 & 0 & 5 \\ 7 & 2 & -5 \\ 3 & 1 & 7 \end{pmatrix}$$

next expand across the top row:

$$3(5(7-6)) = 15$$

3.1-11 Expand across bottom row, then again across bottom row. $3(1(-2(3))) = -18$.

3.1-13 Expand across second row

$$-2\det\begin{pmatrix}4 & 0 & 3 & -5\\ 7 & 3 & 4 & -8\\ 5 & 0 & 2 & -3\\ 0 & 0 & -1 & 2\end{pmatrix}$$

then expand down second column

$$(-2)\cdot 3\cdot\det\begin{pmatrix}4 & 3 & -5\\ 5 & 2 & -3\\ 0 & -1 & 2\end{pmatrix} = (-2)\cdot 3\cdot(4(4-3)-5(6-5)) = 6$$

3.2-1 Switching two rows multiplies the determinant by -1.

3.2-2 Adding a multiple of one row to another doesn't change the determinant.

3.2-21 The determinant is $2(6-18)-6(2-6) = 0$ so the matrix is not invertible.

3.2-22 The determinant is $5(-9+10)-(3+5) \neq 0$ so the matrix is invertible.

3.2-24

$$\det\begin{pmatrix}4 & -7 & -3\\ 6 & 0 & -5\\ 2 & 7 & -2\end{pmatrix} = 0$$

so the vectors are linearly dependent.

3.2-25

$$\det\begin{pmatrix}7 & -8 & 7\\ -4 & 5 & 0\\ -6 & 7 & -5\end{pmatrix} = -1 \neq 0$$

so the vectors are linearly independent.