Math 54 Homework 2 Solutions

1.3-11 Want to solve for c_1, c_2, c_3:

$$c_1 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + c_3 \begin{pmatrix} 5 \\ -6 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

$$\begin{aligned} c_1 + 5c_3 &= 2 \\ -2c_1 + c_2 - 6c_3 &= -1 \\ 2c_2 + 8c_3 &= 6 \end{aligned}$$

can be solved using row reduction. The end result is

$$\left(\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

which has solutions. Therefore $\mathbf{b}$ is in the span of $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$. Explicitly, the solutions are the line satisfying $c_1+5c_3 = 2$ and $c_2+4c_3 = 3$. To get a solution, pick a value for the free variable c_3, say 0. Then $c_1 = 2$ and $c_2 = 3$ so

$$2 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + 3 \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

1.3-12 Similar to the last one, except now the augmented matrix is

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{array}\right)$$

which row reduces to

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right).$$

Further row reduction is possible, but the last row implies that $0 = 2$, which implies there are no solutions (c_1, c_2, c_3) to the equation

$$c_1 \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 5 \\ 5 \end{pmatrix} + c_3 \begin{pmatrix} 2 \\ 0 \\ 8 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ 7 \end{pmatrix}$$

therefore $\mathbf{b}$ is not a linear combination of $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$.

1.3-17 For this you want to solve for c_1 and c_2 satisfying

$$c_1 \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} -2 \\ -3 \\ 7 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ h \end{pmatrix}$$

This involves row reducing

$$\left(\begin{array}{cc|c} 1 & -2 & 4 \\ 4 & -3 & 1 \\ -2 & 7 & h \end{array}\right)$$

which row reduces to

$$\left(\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & h-1 \end{array}\right)$$

at this point one can see this will have a solution only if $h = 1$.

1.3-18 Similar to the last, though with the augmented matrix

$$\left(\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{array}\right)$$

which row reduces to

$$\left(\begin{array}{cc|c} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & 7+2h \end{array}\right)$$

and this only has a solution when $h = -7/2$.

1.3-21 Want to solve

$$a_1 \begin{pmatrix} 2 \\ -1 \end{pmatrix} + a_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix}$$

which means row reducing

$$\left(\begin{array}{cc|c} 2 & 2 & h \\ -1 & 1 & k \end{array}\right)$$

which row reduces to

$$\left(\begin{array}{cc|c} 1 & 0 & \frac{h}{4} - \frac{k}{2} \\ 0 & 1 & \frac{h}{4} + \frac{k}{2} \end{array}\right)$$

so $a_1 = \frac{h}{4} - \frac{k}{2}$ and $a_2 = \frac{h}{4} + \frac{k}{2}$ does the trick.

1.5-17 Parallel planes in $\mathbb{R}^3$

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1.5-19

$$t \mapsto \begin{pmatrix} -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ 3 \end{pmatrix}$$

1.5-20

$$t \mapsto \begin{pmatrix} 3 \\ -4 \end{pmatrix} + t \begin{pmatrix} -7 \\ 8 \end{pmatrix}$$

1.7-1 Want to solve

$$a_1 \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 7 \\ 2 \\ -6 \end{pmatrix} + a_3 \begin{pmatrix} 9 \\ 4 \\ -8 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

If the only solution is $a_1 = a_2 = a_3 = 0$ then they are linearly independent. Otherwise they are not. To solve this system of three equations in three variables, row reduce

$$\left(\begin{array}{ccc|c} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{array}\right)$$

which row reduces to

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

which implies that $a_1 = a_2 = a_3 = 0$, so these vectors are linearly independent.

1.7-2 Similar to the last though with augmented matrix

$$\left(\begin{array}{ccc|c} 0 & 0 & -3 & 0 \\ 0 & 5 & 4 & 0 \\ 2 & -8 & 1 & 0 \end{array}\right)$$

which row reduces to

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right)$$

which implies that $a_1 = a_2 = a_3 = 0$, so these vectors are linearly independent.

1.7-3 The second vector is a multiple of the first, and so they are linearly dependent.

1.7-4 Similar to the previous three problems, though with augmented matrix

$$\left(\begin{array}{cc|c} -1 & -2 & 0 \\ 4 & -8 & 0 \end{array}\right)$$

this row reduces to

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right)$$

making these vectors linearly independent.

1.7-9 (a) $\mathbf{v}_1$ and $\mathbf{v}_2$ are multiples of each other and so lie on the same line. Their span is this line. The only way for $\mathbf{v}_3$ to be in $\mathrm{Span}(\mathbf{v}_1, \mathbf{v}_2)$ is if $\mathbf{v}_3$ is a multiple of $\mathbf{v}_1$ and $\mathbf{v}_2$. Looking at the first two coordinates, this never can be the case.

(b) $\mathbf{v}_1$ and $\mathbf{v}_2$ satisfy the nontrivial linear relation $3\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{0}$. Therefore $\{\mathbf{v}_1, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly dependent for all values of h.

1.7-10 Same as the last problem, except the linear relation between $\mathbf{v}_1$ and $\mathbf{v}_2$ is $2\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{0}$.