Math 54 Homework 1 Solutions

1.1-1

$$\left(\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 5 & 5 \\ 0 & 1 & 3 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right)$$

so the solution is $(-8, 3)$.

1.1-2

$$\left(\begin{array}{cc|c} 2 & 4 & 4 \\ 5 & 7 & 11 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 5 & 7 & 11 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & -3 & 21 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 2 & -2 \\ 0 & 1 & -7 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & -7 \end{array}\right)$$

so the solution is $(12, -7)$.

1.1-3

$$\begin{cases} x_1 + 5x_2 = 7 \\ x_1 - 2x_2 = -2 \end{cases}$$

$$x_1 = 2x_2 - 2 \Rightarrow (2x_2 - 2) + 5x_2 = 7 \Rightarrow 7x_2 = 9 \Rightarrow x_2 = \frac{9}{7}$$

$$\Rightarrow x_1 + \frac{45}{7} = 7 \Rightarrow x_1 = \frac{4}{7}$$

so the answer is $(4/7, 9/7)$.

1.1-4

$$\begin{cases} x_1 - 5x_2 = 1 \\ 3x_1 - 7x_2 = 5 \end{cases}$$

$$x_1 = 5x_2 + 1 \Rightarrow 3(5x_2 + 1) - 7x_2 = 5 \Rightarrow x_2 = \frac{1}{4}$$

$$\Rightarrow 3x_1 - \frac{7}{4} = 5 \Rightarrow x_1 = \frac{9}{4}$$

so the answer is $(9/4, 1/4)$.

1.1-13

$$\left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & -5/2 & 5/2 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & 1 & -1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

so the solution is $(5, 3, -1)$.

1.1-14

$$\left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ -2 & 5 & 7 & \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

so the solution is $(2, -1, 1)$.

1.1-17

$$\left(\begin{array}{cc|c} 1 & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & -7 & 5 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{array}\right)$$

$$\to \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & 0 & -13/7 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{array}\right)$$

so the system

$$\begin{cases} x_1 - 4x_2 = 1 \\ 2x_1 - x_2 = -3 \\ -x_1 - 3x_2 = 4 \end{cases}$$

has the same solution set as

$$\begin{cases} x_1 = -13/7 \\ x_2 = -5/7 \\ 0 = 0 \end{cases}$$

so the two lines intersect at $(-13/7, -5/7)$.

1.1-18

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & -1 & -4 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & -5 \end{array}\right)$$

so the system

$$\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_2 - x_3 = 1 \\ x_1 + 3x_2 = 0 \end{cases}$$

has the same solutions as the system

$$\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_2 - x_3 = 1 \\ 0 = -5 \end{cases}$$

which obviously has no solutions (look at the last equation). Therefore the three planes do not have a common intersection point.

1.1-29 Switch rows 1 and 2.

1.1-30 Multiply row 2 by $-1/2$ and -2.

1.2-3

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{array}\right)$$

$$\to \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

pivot columns are columns 1 and 2.

1.2-4

$$\left(\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} \to 1 & 3 & 5 & 7 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -34 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 17/4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 5/4 \end{array}\right)$$

pivot columns are columns 1 and 2.

1.2-19 If $h = 2$ and $k = 8$ the two equations are multiples of each other and hence define the same line. Therefore there are infinitely many solutions. If $h = 2$ and $k \neq 8$ then the two lines are parallel and hence do not define the same line. If $h = 0$ and $k = 0$ (for example) then there is a solution at $(2, -1)$.

1.2-20 If $h = 9$ and $k = 6$ the two equations are multiples of each other and hence define the same line. Therefore there are infinitely many solutions. If $h = 9$ and $k \neq 6$ then the two lines are parallel and hence do not define the same line. If $h = 0$ and $k = 0$ (for example) then there is a solution at $(0, 2/3)$.