Math 54 Homework 10
Due Tuesday August 13

1. Recall that the heat equation is

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$
$$u(x,0) = f(x)$$

for some initial function f.

   (a) Let $f(x) = e^{2ix} + e^{-2ix} + e^{4ix} + e^{-4ix}$ be the initial condition. If u solves the heat equation with this initial condition, what is $u(0,t)$?

   (b) Let $f(x) = |x|$ on $(-\pi,\pi)$ and repeat f periodically so it becomes a periodic function on $\mathbb{R}$. (You computed the Fourier series for f in homework 9.) Write down a solution to the heat equation with this function f as the initial condition.

2. Recall that the wave equation is

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$
$$u(x,0) = f(x)$$
$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

for some initial functions f and g.

   (a) Write down a solution to the wave equation for $f(x) = \cos(x)$ and let $g(x) = \sin(x)$. (hint: it may be helpful to write each trig function as a sum of two complex exponentials).

   (b) Let $f(x) = 1$ and let $g(x)$ be the periodic function obtained by repeating $g(x) = x$ on the interval $(-\pi,\pi)$. (You computed the Fourier series of g on homework 9.) Write down a solution to the wave equation for this f and g.

3. Suppose u satisfies the following PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u$$
$$u(x,0) = f(x)$$

and that u is periodic with period 2π. Then f is periodic, so can be written as a Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} b_n e^{inx}$$

(a) Write down a formula for $u(x,t)$ in terms of the coefficients b_n.

(b) What happens to u as t goes to infinity?

4. Solve the following PDE

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + 5u$$

$$u(x,0) = f(x)$$

$$\frac{\partial u}{\partial t}(x,0) = g(x)$$

where $f(x) = \cos(3x)$ and $g(x) = \sin(2x) + \sin(x)$. (hint: it may help to write the trig functions in terms of complex exponentials)