Math 110 Midterm 2 (PRACTICE)

July 24, 2018

50 Minutes

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1. Prove that the dot product on $\mathbb{C}^2$

$$(z_1, z_2) \cdot (w_1, w_2) = z_1 w_1 + z_2 w_2$$

is not an inner product.

2. Let V be a finite-dimensional complex inner product space. Prove or disprove: an isometry of V has an eigenbasis.

3. Let T be the transformation of $\mathbb{R}^2$ given, with respect to the standard basis, by

$$\begin{pmatrix} 2 & 1 \\ -2 & -1 \end{pmatrix}.$$

Prove that T is not an orthogonal projection with respect to the dot product on $\mathbb{R}^2$. (Hint: what is the relation between the kernel and image of an orthogonal projection?)

4. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be given, with respect to the standard basis, by the matrix

$$\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

(a) Prove that T is positive semidefinite (with respect to the dot product) and invertible.

(b) Define a new inner product on $\mathbb{R}^2$ by

$$\langle v, w \rangle_T = v \cdot Tw.$$

(On a homework question you already showed that this is an inner product.) Let $\|\cdot\|_T$ be the norm for $\langle \cdot, \cdot \rangle_T$. The norm $\|\cdot\|_T$ gives a new distance function on $\mathbb{R}^2$. Using $\|\cdot\|_T$ to measure distance, find the point on the x-axis closest to $(1, 1)$.