1. In $\mathbb{R}^2$ with the usual distance, find the point on the line $3x + 4y = 0$ closest to the point $(7, 1)$.

2. Let
$$A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}.$$
Show that
$$\langle x, y \rangle := x \cdot Ay$$
is not an inner product on $\mathbb{R}^2$.

3. Let V be a finite-dimensional complex inner product space. A transformation $T : V \to V$ is called skew-adjoint if $T^* = -T$.

   (a) Prove that the eigenvalues of skew-adjoint transformations on V are purely imaginary (i.e., real multiples of i).

   (b) Prove that skew-adjoint transformations on V are diagonalizable.

4. Consider $\mathbb{R}^2$ with the dot product. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be self-adjoint. Prove that
$$\min_{\|v\|=1} \|Tv\|$$
is the absolute value of the smallest eigenvalue of T. (Here $\min_{\|v\|=1} \|Tv\|$ is the smallest value of $\|Tv\|$ on the set of vectors v of norm 1.)