Math 110 Midterm 2

July 26, 2018

50 Minutes

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1. (a) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be given, with respect to the standard basis, by

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

With respect to the dot product on $\mathbb{R}^3$, what is the adjoint of T?

(b) Is TT^* self-adjoint?

2. Let $T : V \to V$ be a normal operator on a complex inner product space whose eigenvalues λ satisfy $|\lambda| < 1$. Prove that $\|Tv\| \leq \|v\|$ for all $v \in V$.

3. (a) Let V be a real inner product space. Prove that

$$\|v + w\|^2 + \|v - w\|^2 = 2(\|v\|^2 + \|w\|^2).$$

(b) Prove that there does not exist an inner product on $\mathbb{R}^2$ such that $\|(a, b)\| = \max(|a|, |b|)$ for $(a, b) \in \mathbb{R}^2$. Here $\max(|a|, |b|)$ means the larger of $|a|$ or $|b|$.

4. Consider $\mathbb{R}^4$ with the dot product. Let U be the span of

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}.$$

What is the orthogonal projection of $(1, 2, 1, 4)$ onto U?