Math 110 Midterm 1

July 10, 2018

50 Minutes

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1. (10 points) Let $T : V \to W$ and $S : W \to U$ be linear maps. Prove that $S \circ T = 0$ if and only if $\text{im}(T) \subset \ker(S)$.

2. (10 points) Let $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$ be elements of $\mathbb{F}$. Suppose that the following n vectors in $\mathbb{F}^n$ are linearly independent:

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix}, \begin{pmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \\ \vdots \\ x_n^2 \end{pmatrix}, \begin{pmatrix} x_1^3 \\ x_2^3 \\ x_3^3 \\ \vdots \\ x_n^3 \end{pmatrix}, \ldots, \begin{pmatrix} x_1^{n-1} \\ x_2^{n-1} \\ x_3^{n-1} \\ \vdots \\ x_n^{n-1} \end{pmatrix}.$$

Prove that there exists a unique polynomial p of degree less than or equal to $n-1$ with coefficients in $\mathbb{F}$ such that $p(x_i) = y_i$ for all i. (Hint: use the vector $(y_1, y_2, \ldots, y_n)$ as well as the above n vectors.)

3. (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map represented, with respect to the standard basis vectors, by the matrix

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Prove or disprove: T is diagonalizable.

4. (10 points) Let $T : V \to V$ be a linear map. Prove or disprove: $V = \ker(T) \oplus \operatorname{im}(T)$.