Math 110 Homework 6
Due Tuesday August 7, 2018

1. Compute the determinant of

$$
\begin{pmatrix}
2 & -1 & 0 & 0 & \cdots & 0 & 0 \\
-1 & 2 & -1 & 0 & \cdots & 0 & 0 \\
0 & -1 & 2 & -1 & \cdots & 0 & 0 \\
0 & 0 & -1 & 2 & \ddots & 0 & 0 \\
0 & 0 & 0 & -1 & \ddots & -1 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & 2 & -1 \\
0 & 0 & 0 & \cdots & 0 & -1 & 2
\end{pmatrix}.
$$

   (2s on the main diagonal, and -1 on each diagonal above and below that).

2. (a) Prove that $M_{\sigma(1)1}M_{\sigma(2)2}\cdots M_{\sigma(n)n} = M_{1\sigma^{-1}(1)}M_{2\sigma^{-1}(2)}\cdots M_{n\sigma^{-1}(n)}$.

   (b) Prove that

   $$
   \sum_{\sigma} M_{\sigma(1)1}M_{\sigma(2)2}\cdots M_{\sigma(n)n} = \sum_{\sigma} M_{\sigma^{-1}(1)1}M_{\sigma^{-1}(2)2}\cdots M_{\sigma^{-1}(n)n}.
   $$

   (c) Prove that $\det(M) = \det(M^{\top})$.

3. Let M be a real $n \times n$ matrix with orthonormal columns (with respect to the dot product). Prove that $\det(M) = \pm 1$.

4. Find the characteristic polynomial and eigenvalues of

$$
\begin{pmatrix}
6 & 6 & 1 \\
-1 & -1 & -1 \\
-6 & -6 & -1
\end{pmatrix}.
$$

5. Let A be a complex matrix. Express the coefficients of the characteristic polynomial of A in terms of the eigenvalues of A.