Math 110 Final (PRACTICE: 3/4 length)

August 7, 2018

80 Minutes

Name:________________________

1	
2	
3	
4	
5	
6	

1. What are the eigenvectors and eigenvalues of

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}?$$

2. Let $T : V \to V$ be a linear map with $\dim(\operatorname{im}(T)) = 1$. Prove that T has an eigenvector.

3. Let V be an inner product space. Let $\{v_1, \ldots, v_m\}$ be a collection of vectors such that

$$\langle v_i, v_j \rangle = \begin{cases} 2 & i = j \\ -1 & i \neq j \end{cases}.$$

Prove that any two distinct vectors $\{v_i, v_j\}$ are linearly independent, but any three distinct vectors $\{v_i, v_j, v_k\}$ are linearly dependent.

4. Let V be the inner product space of continuous functions $f : [-1,1] \to \mathbb{R}$ with the L^2 inner product:

$$\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)dx.$$

Let U be the span of the functions $f(x) = 1$ and $f(x) = x^2$. What is the orthogonal projection of $f(x) = x^4$ to U? (The final answer involves some fractions, but no radicals.)

5. Do any three four-dimensional subspaces of $\mathbb{R}^5$ contain a line in their intersection? Explain.

6. The space $\mathcal{M}_{n,k}$ of multilinear maps

$$\phi : \underbrace{\mathbb{F}^n \times \mathbb{F}^n \times \cdots \times \mathbb{F}^n}_{k \text{ times}} \to \mathbb{F}$$

forms a vector space in the usual way

$$(\phi + \psi)(v_1, \ldots, v_k) := \phi(v_1, \ldots, v_k) + \psi(v_1, \ldots, v_k)$$

$$(c\phi)(v_1, \ldots, v_k) = c\phi(v_1, \ldots, v_k).$$

In terms of n and k, what is $\dim(\mathcal{M}_{n,k})$? Justify your answer.