Math 110

July 31, 2018

Singular Value Decomposition 2

1. Consider $\mathbb{R}^2$ with the dot product. Find the singular vectors and singular values of the transformation given, with respect to the standard basis, by
$$\begin{pmatrix} 2 & -2 \\ 1 & 1 \end{pmatrix}.$$
Graph the singular vectors on the plane. Show the effect to the transformation on the singular vectors for the domain v_1 and v_2.

2. Consider $\mathbb{R}^2$ with the dot product. Find the singular vectors and singular values of the transformation given, with respect to the standard basis, by
$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}.$$
Graph the singular vectors on the plane. Show the effect to the transformation on the singular vectors for the domain v_1 and v_2.

3. Consider $\mathbb{R}^2$ with the dot product. Find the singular vectors and singular values of the transformation given, with respect to the standard basis, by
$$\begin{pmatrix} 2 & 2 \\ 1 & 1 \end{pmatrix}.$$
Graph the singular vectors on the plane. Show the effect to the transformation on the singular vectors for the domain v_1 and v_2.

4. Consider $\mathbb{R}^2$ and $\mathbb{R}^3$ with the dot product. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be given, with respect to the standard basis, by the matrix
$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$
Find the singular vectors and singular values of T. You can use the fact that the eigenvalues of
$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$
are 3, 1, and 0.

5. Let $(v_1, \ldots, v_n)$, $(w_1, \ldots, w_m)$, and $(s_1, \ldots, s_{\min(m,n)})$ be the singular vectors and values of a transformation $T : V \to W$. What are the singular vectors and values of $T^* : W \to V$? (hint: $\langle Tv_i, w_j \rangle = \langle v_i, T^* w_j \rangle$)