Math 110

June 28, 2018

More Eigenstuff

1. Think of
$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$
as a linear map $\mathbb{C}^2 \to \mathbb{C}^2$. Find its eigenvectors and eigenvalues.

2. Let $P : V \to V$ a map of finite-dimensional vector spaces. Prove that the following are equivalent:
   (a) $P^2 = P$
   (b) P is diagonalizable and its eigenvalues belong to the set $\{0, 1\}$
   (c) There exist subspaces U, W such that $V = U \oplus W$ and P is the projection onto U (i.e., if $v = u + w$ for $u \in U$ and $w \in W$ then $P(v) = u$).

3. Is the following true or false? Why?

   "Given $T : V \to V$ and $U \subset V$ an invariant subspace, there exists another invariant subspace W such that $V = U \oplus W$".