Math 110

June 21, 2018

Bases and Linear Transformations

1. Let $\{v_1, v_2, \ldots\}$ be a linearly independent collection of vectors. Show that any vector in $\text{Span}\{v_1, v_2, \ldots\}$ can be uniquely expressed as a linear combination of the v_is.

2. Let $\{v_1, v_2, \ldots, v_n\}$ be a finite spanning set for a vector space V. Show that $\{v_1, \ldots, v_n\}$ contains a basis for V. (hint: remove "unecessary" vectors one by one until you end up with a linearly independent set)

3. Let V be a finite-dimensional vector space and let $\{v_1, v_2, \ldots, v_n\}$ be a linearly independent set of vectors. Show that you can add vectors to $\{v_1, v_2, \ldots, v_n\}$ to produce a basis.

4. In high school, you might have defined a "linear function" $\mathbb{R} \to \mathbb{R}$ as one of the form $f(x) = ax + b$. When is such function linear in the sense of linear algebra?

5. Let $T : V \to W$ be linear. Show that $T(0) = 0$.

6. Let $T : V \to W$ be an isomorphism. Let T^{-1} be the inverse. Show that T^{-1} is linear.

7. Prove that if $T : V \to W$ is an isomorphism and $(v_1, v_2, \ldots)$ a basis of V then $(T(v_1), T(v_2), \ldots)$ is a basis of W.

8. Show that differentiation of differentiable functions is a linear map. Is it invertible? Is integration of continuous functions a linear map?