Math 110

June 19, 2018

Vector Spaces, Linear Independence, Subspaces, Span

1. Graph the vectors
$$\begin{pmatrix} -1 \\ -3 \end{pmatrix} \text{ and } \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
in $\mathbb{R}^2$. Graph their sum by gluing them end to end.

2. Are the vectors
$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
linearly independent in $\mathbb{F}^2$?

3. Are the vectors
$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
linearly independent in $\mathbb{F}^2$?

4. Let V be the vector space of polynomials over $\mathbb{F}$. Are the monomials x^n and x^m linearly independent in V?

5. Prove that the subset of $\mathbb{F}^n$ containing vectors of the form $(a_1, \ldots, a_{n-1}, 0)$ is a subspace. Prove that the subset of $\mathbb{F}^n$ containing vectors of the form $(a_1, \ldots, a_{n-1}, 1)$ is not.

6. Prove that the intersection of two subspaces is a subspace.

7. Let $W \subset V$ be a subspace. Prove that if $w \in W$ then $-w \in W$. Here $-w$ denotes the additive inverse of w.

8. In class, I said a sum $W_1 + W_2$ is direct if $W_1 \cap W_2 = \{0\}$. Axler defines it a little differently. He says that a sum $W_1 + W_2$ is direct if each element $w_1 + w_2 \in W_1 + W_2$ (where $w_i \in W_i$) can be written uniquely. That is, if $w_1 + w_2 = w_1' + w_2'$ then $w_1 = w_1'$ and $w_2 = w_2'$. Prove that these two definitions are equivalent. (Two definitions are equivalent if each implies the other.)

9. Are
$$\text{Span}\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\} \text{ and } \text{Span}\left\{ \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$$
the same subspace of $\mathbb{F}^3$?

10. Remove the axiom $1v = v$ from the list of axioms of a vector space. Show that the set $\mathbb{F}^n$ with addition given by

$$(a_1, \ldots, a_n) + (b_1, \ldots, b_n) := (a_1 + b_1, \ldots, a_n + b_n)$$

and scalar multiplication given by

$$c(a_1, \ldots, a_n) := (0, \ldots, 0)$$

satisfies the remaining axioms. Prove that the axiom $1v = v$ is not implied by the other axioms.