

Math 110

June 18, 2018

Reacquaintance With Linear Algebra

1. Let

$$A = \begin{pmatrix} 3 & 4 \\ 1 & -2 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, C = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

Find AB and BC . What is the relation between $(AB)C$ and $A(BC)$?

2. How does the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ act on the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ in \mathbb{R}^2 ?

3. How might you indicate, geometrically/visually, how the following matrices act on \mathbb{R}^2 ?

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

4. Solve

$$\begin{cases} 2x + y = 0 \\ x + y = 0 \end{cases}, \begin{cases} 2x + y = 5 \\ x + y = -1 \end{cases}, \begin{cases} 3x + 2y + z = 3 \\ -2x + y = 0 \\ -x + 4y + z = 3 \end{cases}$$

5. Write the above three systems of equations in terms of $Ax = b$ where A is a matrix and x and b are vectors.
6. What is the relation between row reduction and solving systems of equations?
7. Diagonalize:

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

(this is partly a trick question)

8. Graph the line

$$t \mapsto \begin{pmatrix} t + 3 \\ 2t + 1 \end{pmatrix}$$

and find the point on it closest to the origin.

9. Find the orthogonal projection of $(1, -6, 4)$ onto the subspace spanned by $(1, 2, 3)$ and $(1, 0, 0)$.

10. Find $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^n$.