Math 10a
Practice Midterm 1 #1

1.) a.) $\boxed{10m^t \text{ L}}$

b.) $10m^{100} = 2\cdot 10$

$m^{100} = 2$

$m = \sqrt[100]{2} = \boxed{2^{1/100}}$

c.) $10m^t = 50$

$m^t = 5$

$(2^{1/100})^t = 5$

$2^{t/100} = 5$

$\frac{t}{100}\ln(2) = \ln(5)$

$t = \boxed{\frac{100\ln(5)}{\ln(2)} \text{ seconds after start}}$

d.) $V(t) = 10m^t$

$= 10\cdot 2^{t/100}$

$\frac{dV}{dt} = 10\cdot 2^{t/100}\cdot\frac{1}{100}\cdot\ln(2)$

$\boxed{\left.\frac{dV}{dt}\right|_{t=0} = \frac{\ln 2}{10}}$

2.) (a) $f(x) = \frac{1}{(1+x^2)}$

$f'(x) = \frac{-2x}{(1+x^2)^2}$

(b) $\sqrt{\sqrt{x}} = \sqrt[4]{x}$

$f(x) = \sqrt[4]{x}$

$f'(x) = \frac{1}{4}x^{-3/4}$

(c) $f(x) = \cos^2\left(\frac{x^2}{2}\right)$

$f'(x) = 2\cos\left(\frac{x^2}{2}\right)\cdot -\sin\left(\frac{x^2}{2}\right)\cdot(x)$

$= -2x\cos\left(\frac{x^2}{2}\right)\sin\left(\frac{x^2}{2}\right)$

2(c+d)

(d) $f(x) = \ln(1+\sqrt{x})$

$f'(x) = \frac{1}{1+\sqrt{x}}\cdot\frac{1}{2}x^{-\frac{1}{2}}$

$= \frac{1}{2}\cdot\frac{1}{\sqrt{x}+x}$

3.) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Differentiating implicitly,

$\frac{2x}{4} - \frac{2y}{9}\frac{dy}{dx} = 0$

at $(2\sqrt{2}, 3)$ this gives

$\sqrt{2} - \frac{2}{3}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{3\sqrt{2}}{2}$

tangent line

$\boxed{y - 3 = \frac{3\sqrt{2}}{2}(x - 2\sqrt{2})}$

4.) $\sqrt[3]{2}$ is a root of $f(x) = x^3 - 2$, so apply Newton's method to $f(x) = x^3 - 2$ with guess $x_1 = \frac{5}{4}$

$f'(x) = 3x^2$

$x_2 = x_1 - \frac{(x_1)^3 - 2}{3(x_1)^2}$

$= \frac{5}{4} - \frac{(\frac{5}{4})^3 - 2}{3(\frac{5}{4})^2} = \frac{5}{4} - \frac{\frac{125}{64} - \frac{128}{64}}{3(\frac{25}{16})}$

$= \frac{5}{4} + \frac{\frac{3}{64}}{3\cdot\frac{25}{16}} = \frac{5}{4} + \frac{1}{100} = \frac{63}{50}$

5. a) $f(x) = \sin(x)$ $\qquad a = 0$

$f(0) = 0$

$f'(x) = \cos(x) \qquad f'(0) = 1$

$f''(x) = -\sin(x) \qquad f''(0) = 0$

$f'''(x) = -\cos(x) \qquad f'''(0) = -1$

$$T_3(x) = 0 + \frac{1}{1!}(x-0) + \frac{0}{2!}(x-0)^2 + \frac{-1}{3!}(x-0)^3$$

$$= x - \frac{x^3}{6}$$

b.) $\sin\left(\frac{1}{4}\right) \approx \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{6} = \frac{1}{4} - \frac{1}{384} = \frac{95}{384}$

Math 10a

Practice Midterm 1 #2

1.) $f(x) = \frac{1}{1+x^2}$

a. $1+x^2$ takes any value in $[1,\infty)$, so $\frac{1}{1+x^2}$ has range $(0,1]$.

b.) Consider x on the domain $[0,\infty)$

then $y = \frac{1}{1+x^2}$

$$1+x^2 = \frac{1}{y}$$

$$x^2 = \frac{1}{y} - 1$$

$$x = \sqrt{\frac{1}{y}-1}$$

$$f^{-1}(y) = \sqrt{y^{-1}-1}$$

2.) a) $\lim_{x\to\infty} \frac{3x^2}{x^2+1} = \lim_{x\to\infty} \frac{3}{1+\frac{1}{x^2}} = 3$

b) $\lim_{x\to 0} \frac{1}{x}$ does not exist

$$\infty = \lim_{x\to 0^+} \frac{1}{x} \neq \lim_{x\to 0^-} \frac{1}{x} = -\infty$$

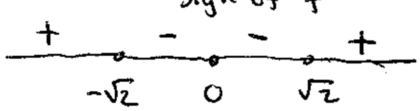
c.) $\lim_{x\to 0} \frac{x}{x+3} = \frac{0}{0+3} = 0$

3.) $f(x) = 6x^5 - 20x^3$

a.) $f'(x) = 30x^4 - 60x^2$
$= 30x^2(x^2-2)$

0 at $x = 0, \pm\sqrt{2}$

sign of f'



$f'(1) = f'(1) = -30 < 0$

$f'(2) = f'(-2) = 240$

f is increasing on $(-\infty,-\sqrt{2})$ and $(\sqrt{2},\infty)$

decreasing on $(-\sqrt{2},\sqrt{2})$

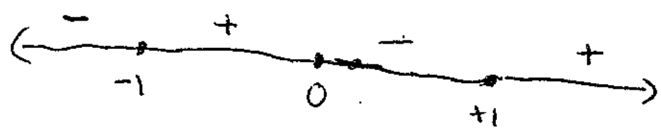
3 (ctd)

b.) by (a) local max at $(-\sqrt{2}, 16\sqrt{2})$
local min at $(\sqrt{2}, -16\sqrt{2})$

c.) $f''(x) = 120x^3 - 120x$

$f''(x) = 120x(x+1)(x-1)$

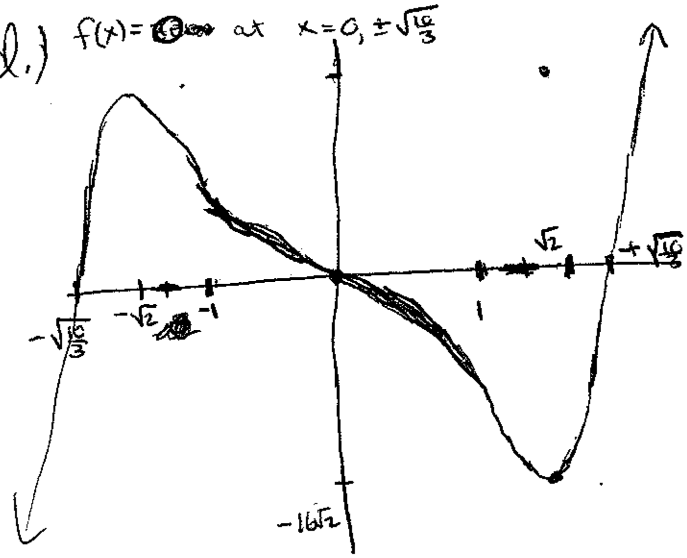
sign of $f''(x)$



Concave up on $(-1,0)$, $(1,\infty)$

concave down on $(-\infty,-1)$, $(0,1)$

d.) $f(x) = 0$ at $x = 0, \pm\sqrt{\frac{10}{3}}$



(note different scales on x- and y- axes)

4) a) $P'(t) = kP(M-P)$

b) Take P derivative of $P'(t)$

$$\frac{dP'}{dP} = kP(-1) + k(M-P)$$

$$= k(M-2P)$$

0 at $P = \frac{1}{2}M$

Population grows fastest when ~~it~~ it is $\frac{1}{2}M$

5. $$\sum_{n=0}^{\infty} \frac{3 \cdot 3^{2n}}{4^{2n}} = 3 \sum_{n=0}^{\infty} \frac{(3^2)^n}{(4^2)^n} = 3 \sum_{n=0}^{\infty} \left(\frac{9}{16}\right)^n$$

geometric series with $r = \frac{9}{16} < 1$

$$= 3 \frac{1}{1-\frac{9}{16}} = 3 \cdot \frac{1}{\frac{7}{16}} = \frac{48}{7}$$