Math 10a

October 14, 2014

u-Substitutions and Some Integral Miscellania

1.
$$\int \sin(x)\cos(x)dx,\ \int e^{a+bx}dx,\ \int \frac{\ln(x)}{x}dx,\ \int x\sin(x^2)dx,\ \int \frac{x+1}{\sqrt{x-1}}dx,\ \int \tan(x)dx$$

2.
$$\int_2^3 \frac{x}{1-x}dx,\ \int_2^3 \frac{\ln(\ln(x))}{x}dx,\ \int_0^1 x\sqrt{1-x}dx,\ \int_0^\pi \sin^3(\theta)d\theta$$

3.
$$\int \frac{x^2}{1+x^2}dx \text{ (hint: add and subtract something to the numerator)}$$

4. What do you think $\int_0^\infty e^{-t}dt$ is?

5. What do you think $\int_1^\infty \frac{1}{x}dx$ is?

6. A toy car's velocity, as a function of time, is $3\sin(2t)$. If it starts at an initial position of $x = 0$, what is position of the car as a function of time? If a kid is playing with the car, what do you think he's doing to it?

7. Consider 12 inch metal rod whose density x inches from the end is $\frac{1}{2}x(12-x)$ kg/in. What is the total mass of the rod?

8. At a particular point in the desert, the intensity $I(t)$ of sunlight changes at a rate of
$$5\cos\left(\frac{\pi t}{24}\right)\sin\left(\frac{\pi t}{24}\right)$$
where t is measured in hours. If the intensity at time $t = 0$ is 2, then what is the intensity as a function of time?

9. An electron in the lowest energy state of a hydrogen atom does not exist at a single point but rather there is a chance it is observed at any distance from the atom. The chance that it is found at a radius between r and $r + dr$ (for small dr) is
$$\frac{2}{a}e^{-2r/a}dr$$
where $a = 5.29 \times 10^{-9}$ m. What is the chance that the electron is observed between 5×10^{-9} and 6×10^{-9} meters from the hydrogen atom?