

Math 10a
September 16, 2014
Derivatives

1. Use the limit definition of the derivative to compute the derivative of \sqrt{x} at $x = 5$. (hint: you may need to multiply the top and bottom of the fraction by something)
2. For each of the following, functions, write it as a composition $f \circ g$ for some functions f and g . Compute the derivatives of f and g , and then find the derivative of $f \circ g$.

$$e^{x^2}, (x+1)^3, \ln(\sqrt{1+x}), \ln(\ln(x)), (\ln(x))^2, (x^2+6x+7)^9.$$

3. Is $e^{\cos(x)}$ an increasing function?
4. Differentiate the following with the quotient rule:

$$\frac{1}{x-1}, \frac{1}{(x-1)^2}, \frac{x-2}{x-3}, \frac{\ln(\ln(x))}{x^2+2x+3}.$$

5. Compute the second derivatives of

$$e^{x^2}, \cos(\sqrt{x}), (x^2+5x+1)^3$$

6. Find a parabola with y -intercept $(0, 6)$, x -intercept $(4, 0)$ and such that the slope of its tangent line as it crosses the x -axis is -3 (hint: write $ax^2 + bx + c$ and solve for a , b , and c .)
7. The second derivative of a parabola at its vertex is negative. What can you say about the parabola?
8. Let $f(x) = x^3 - x$. What is the equation of the tangent line to f at $x = 0$?
9. Let $f(x) = \cos(x)$. What is the equation of the tangent line to f at $x = \frac{\pi}{4}$? (hint: if a line goes through a point (a, b) with slope m , then all points (x, y) on the line satisfy $\frac{y-b}{x-a} = m$ —this is just the definition of the slope—so the equation of the line is $y - b = m(x - a)$.)
10. Let $f(x) = (x+4)^3$. What is the equation of the tangent line to f at $(-2, 9)$?
11. In each of the following, find an equation for $\frac{dy}{dx}$ in terms of x and y .

$$x^2 + y^2 = 1, y^2 = x^3 - x^2, \frac{1}{\sqrt{y}} + \frac{1}{\sqrt{x}} = 1.$$

12. Using the fact that $\tan(\arctan(x)) = x$, show that the derivative of $\arctan(x)$ is $\frac{1}{1+x^2}$.
13. Suppose $y^2 = x^3 - x + 1$. Find the points where the $y' = 0$. Use this information, plus intercepts, to sketch a graph of the curve satisfying this relation.