Name:

Math 10a
October 23, 2014
Quiz #6

1. (2 points) Write down, in summation notation, the estimate for

$$\int_{-1}^{1} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

using a left Riemann sum with 100 intervals.

$$\frac{2}{100} \sum_{k=0}^{99} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(-1+\frac{2k}{100}\right)^2}$$

2. (3 points) If you were to estimate the area under the graph of $\sin(x)$ from 0 to π using the trapezoid rule, how many trapezoids would you need to guarantee accuary to within 1/10? You may use:

$$\text{error}(T_n) \leq \frac{(\max|f''|)(b-a)^3}{12n^2}$$

and the fact that $\pi^3 = 31.00627\ldots$.

The largest value of the (absolute value of the) second derivative is 1. Hence an upper bound for the error is

$$\frac{\pi^3}{12n^2} \approx \frac{31}{12n^2}.$$

In order to guarantee this upper bound to be less than $\frac{1}{10}$, we need to pick n such that

$$\frac{31}{12n^2} < \frac{1}{10} \Rightarrow n^2 > \frac{155}{6} = 25 + \frac{5}{6}$$

so we need $n = 6$.

3. Consider $f(x) = x^2 - x$

(a) (1 point) Sketch the graph of f from $x = -1$ to $x = 1$.

(b) (1 point) Draw the five rectangles of width 2/5 which approximate the area under the curve using the midpoint rule.

(c) (3 points) Approximate

$$\int_{-1}^{1} (x - x^2)dx$$

using a midpoint rule with five intervals. Simplify your answer to a single fraction.'

$$\frac{2}{5}\left(f(-4/5) + f(-2/5) + f(0) + f(2/5) + f(4/5)\right) = -\frac{16}{25}.$$