Main Topic: Related Rates

1. A pole of length 10 feet rests against a vertical wall. If the bottom of the pole slides away from the wall at a speed of 2 ft/s, how fast is the angle between the top of the pole and the wall changing when the angle is $\frac{\pi}{4}$ radians?

   $\theta = \angle$ between top and the wall.
   $\sin \theta = \frac{4}{10}$

   $\cos \theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$

   $\frac{1}{2} \frac{d\theta}{dt} = \frac{1}{10} \cdot 2$

   $\frac{d\theta}{dt} = \frac{2}{5\sqrt{2}} = \frac{\sqrt{2}}{5}$ radians/second.

2. A girl flies a kite at a height of 20 ft, the wind carrying the kite horizontally away from her at a rate of 10 ft per second. How fast must she let the string out when the kite is 40 ft away from her?

   $x$: distance of kite horizontally away from the girl $\frac{dx}{dt} = 10$ ft/s

   $y$: distance of kite away from the girl (string length) $\frac{dy}{dt} = ?$ y = 40 ft.

   $x^2 + 20^2 = y^2$

   $2x \frac{dx}{dt} = 2y \frac{dy}{dt}$

   $\frac{10}{40}$

   not directly given in the problem, but since $x^2 + 20^2 = y^2$, where $y = 40$

   $x = \sqrt{40^2 - 20^2} = \sqrt{1200}$

   $\Rightarrow \frac{dy}{dt} = \frac{2 \cdot \sqrt{1200} \cdot 10}{2 \cdot 40} = \frac{\sqrt{1200}}{4} = \frac{20\sqrt{3}}{4} = 5\sqrt{3}$ ft/s
3. (from 2013 midterm2) A cylindrical container of radius $R$ is being filled up with water at a rate of 10 cubic meters per hour. At what rate is the height of the water increasing? Your solution should appear as a function of $R$.

\[ V = \pi R^2 h \]
\[ \frac{dV}{dt} = \pi R^2 \frac{dh}{dt} = 10 \]
\[ \Rightarrow \frac{dh}{dt} = \frac{10}{\pi R^2} \]

4. (from HW7) A spherical balloon is inflated with helium at the rate of $100\pi$ ft$^3$/min. How fast is the balloon's radius increasing at the instant the radius is 5 ft? How fast is the balloon's surface area increasing at the instant the radius is 5 ft?

\[ V = \frac{4}{3} \pi r^3 \]
\[ \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \]
\[ 100\pi = 4\pi \cdot 5^2 \frac{dr}{dt} \]
\[ \Rightarrow \frac{dr}{dt} = 1 \text{ ft/s} \]

\[ S = \text{surface area of balloon} \]
\[ S = 4\pi r^2 \]
\[ \frac{dS}{dt} = 4\pi \cdot 2r \frac{dr}{dt} \]
\[ \frac{dS}{dt} = 4\pi \cdot 2 \cdot 5 \cdot 1 = 40\pi \text{ ft}^2/\text{s} \]