1. Find the area in 1st quadrant that is bounded above by \( y = \sqrt{x} \) and below by \( x \) axis and \( y = x - 6 \).

Intersection point: \((9, 3)\)

\[
\frac{\sqrt{x}}{x} = \frac{x}{x} - 6
\]

Solve: \( x = 9 \) 

\[
\text{Approach 1: } \int_{0}^{9} \sqrt{x} \, dx
\]

\[
= \frac{2}{3} x^{3/2} \bigg|_{0}^{9}
= \frac{2}{3} (9)^{3/2} - \frac{2}{3} (0)^{3/2}
= \frac{2}{3} (27) - \frac{2}{3} (0)
= 18
\]

\[
\text{Approach 2: } \int_{x=0}^{x=9} (x - 6) \, dx
\]

\[
= \frac{1}{2} x^2 \bigg|_{0}^{9} - 6x \bigg|_{0}^{9}
= \frac{1}{2} (9^2) - 6(9) - \frac{1}{2} (0^2) - 6(0)
= \frac{81}{2} - 54
= 18
\]

2. The region underneath \( y = \sqrt{R^2 - x^2} \) is revolved around the \( x \) axis. Compute the volume. \((R \) is some constant.)

\[
\int_{-R}^{R} \pi (R^2 - x^2) \, dx
\]

\[
= \pi \left[ \frac{1}{3} x^3 \right]_{-R}^{R} - \pi \left[ \frac{1}{3} x^3 \right]_{0}^{R}
= \pi \left[ \frac{2}{3} R^3 - \frac{1}{3} R^3 \right]
= \frac{2}{3} \pi R^3
\]

This is how the formula of volume for sphere is derived.

3. Consider the region bounded by \( y = x \) and \( y = x^2 \), \( 0 \leq x \leq 1 \).

(a) Compute its area
(b) Rotate this region around \( x \) axis. What's the volume?

(c) [Optional] Rotate this region around the line \( x = -1 \). What is the volume?

<table>
<thead>
<tr>
<th>(a) ( \int_{0}^{1} x - x^2 , dx )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{6} )</td>
</tr>
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</table>

<table>
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<tr>
<th>(b) ( \int_{0}^{1} \pi x^2 , dx - \int_{0}^{1} \pi (x^2)^2 , dx )</th>
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</thead>
<tbody>
<tr>
<td>( \pi \left[ \frac{1}{3} x^3 \right]<em>{0}^{1} - \pi \left[ \frac{1}{5} x^5 \right]</em>{0}^{1} )</td>
</tr>
<tr>
<td>( \frac{1}{3} \pi - \frac{1}{5} \pi = \frac{2}{15} \pi )</td>
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<th>(c) [Optional] ( y = x \Rightarrow x = y ) [inner curve]</th>
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<tr>
<td>( y = x^2 \Rightarrow x = \sqrt{y} ) [outer curve]</td>
</tr>
</tbody>
</table>

\[
V = V_{\text{outer}} - V_{\text{hole}}
\]

\[
= \int_{0}^{1} \pi (1+y)^2 \, dy - \int_{0}^{1} \pi (1+y)^2 \, dy
= \pi \left[ \frac{1}{2} y^2 + y - \frac{1}{2} y^3 \right]_{0}^{1}
= \pi \left( \frac{3}{2} - \frac{1}{2} \right) = \frac{1}{2} \pi
\]
4. \( R \) is the region below the curve \( y = \sqrt{1 - x} \) between \( x = 0 \), \( x = 1 \), and above the \( x \)-axis. Compute the volume of the solid obtained by rotation of the region \( R \) around \( y \)-axis.

\[
y = \sqrt{1 - x} \quad \Rightarrow \quad y^2 = 1 - x \\
\chi = 1 - y^2 \\
\int_0^1 \pi (1 - y^2)^2 \, dy \\
= \int_0^1 \pi (1 - 2y^2 + y^4) \, dy \\
= \pi \left( \frac{y - \frac{2y^3}{3} + \frac{y^5}{5}}{1} \right) \bigg|_{y=1}^{y=0} \\
= \pi \left( 1 - \frac{2}{3} + \frac{1}{5} \right) \\
= \frac{8}{15} \pi
\]