Steps for optimization problems: (2 keys!)

1. Draw & label
2. Constraint
3. Formula for the something we want to optimize
4. Domain
5. Find global max/min & justify by table of $f'$.

1. Above steps must be quite abstract. Let's look at one easier example. (problem from textbook)
   Which rectangle of area $100$ in$^2$ minimizes its height plus two times its length?

\[
\begin{align*}
\text{Constraint:} & \quad hl = 100 \\
\Rightarrow h & = \frac{100}{l} \\
\text{Formula:} & \quad S = h + 2l \\
S' & = -\frac{100}{l^2} + 2 = 0 \quad \Rightarrow \text{critical pts: } l = \sqrt{50} \\
S & = \frac{100}{\sqrt{50}} + 2\sqrt{50} \\
\end{align*}
\]

2. It's time for you to try! (from midterm of Fall 2014 Lec003)
   A farmer wants to fence an area of $15,000,000$ square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Find the dimensions of the fenced area that will minimize the amount of the fencing material used.

\[
\begin{align*}
\text{Constraint:} & \quad xy = 15,000,000 \\
\Rightarrow y & = \frac{15,000,000}{x} \\
\text{Formula:} & \quad M = 3x + 2y \\
M' & = 3 - \frac{30,000,000}{x^2} \quad (x>0) \\
\Rightarrow \text{critical pts: } & \quad x = 1000 \\
y & = \frac{15,000,000}{1000} = 1500 \\
M' & = -\frac{1}{1000} \Rightarrow x \\
\text{When height is 1000 ft, width is 1500 ft, the material is minimized.}
\end{align*}
\]
Properties of Exponentials and logarithms:

1. \[ e^{a+b} = e^a e^b \]
2. \[ \ln(a + b) = \ln(a) + \ln(b) \]
3. \[ \ln(a b) = \ln(a) + \ln(b) \]

3. Limit computing involving exponentials and logarithms

(these following problems are modified from our textbook; similar problems appear in hw8 as well)

(a) \[ \lim_{x \to \infty} \frac{\arctan(x+1)}{\ln x} \]

(b) \[ \lim_{x \to \infty} x^3 e^{-x} \]

(c) \[ \lim_{x \to \infty} \frac{x^4 e^{-x}}{e^{-x}} \]

(d) \[ \lim_{x \to \infty} \frac{e^x - x}{x^2} \]

(e) \[ \lim_{x \to \infty} \frac{\ln(x^2)}{\ln x} \]

(f) \[ \lim_{x \to \infty} \ln(x+2) - \ln x \]

(g) \[ \lim_{x \to \infty} (x + x!) \]

(a) "Plug in": \[ \frac{0}{0} \] \Rightarrow \text{L'Hopital}

? = \[ \lim_{x \to \infty} \frac{1 + (x+1)^2}{x} \]

(b) "Plug in": \[ \frac{0}{0} \] \Rightarrow \text{L'Hopital}

? = \[ \lim_{x \to \infty} \frac{1}{x} \left( e^{x+1} \right) \]

(c) Plug in \[ \frac{0}{0} \]

highest order: \[ e^t \]

\[ \frac{1}{e^t} \]

\[ \lim_{t \to \infty} \frac{1}{t} \]

\[ \frac{1}{e^t} \]

? = \[ \frac{1}{2} \]

(d) "Plug in": \[ \frac{0}{0} \]

\[ \lim_{x \to \infty} \frac{\ln x}{x} \]

? = \[ 0 \]

(e) "Plug in": \[ \frac{0}{0} \]

Can simplify!

? = \[ \lim_{x \to \infty} \frac{2 \ln x}{\ln x} \]

? = \[ 2 \]

(f) \[ \lim_{x \to \infty} \frac{\ln(x+2)}{x} \]

? = \[ 0 \]

(g) \[ \lim_{x \to \infty} \frac{\ln(x+1)}{x+1} \]

\[ \lim_{x \to \infty} \frac{1}{x+1} \]

? = \[ 0 \] \Rightarrow \[ L = e^0 = 1 \]

(h) \[ \lim_{x \to \infty} \frac{x e^x}{x^2 e^x} \]

(i) \[ \lim_{x \to \infty} \frac{1 - \cos x}{\ln x} \]

(h) "\[ \frac{0}{0} \]"

highest order: \[ xe^x \]

\[ \frac{x^2 e^x}{x^2 e^x} \]

\[ \frac{1}{x e^x} \]

? = \[ 0 \]

(i) "\[ \frac{0}{0} \]"

L'Hopital

? = \[ \lim_{x \to \infty} \frac{\sin x}{(x+1) e^x + x e^x} \]

? = \[ 0 \]

\[ \frac{1}{x+1} \]

\[ e^x \]

\[ 1 + 0 \]

\[ = 0 \]