

```
In[2]:= P[n_] := With[{S = n (n + 1) / 2},
  NestList[Table[Total[#][[Max[1, s - n] ;; s - 1]], {s, S}] &,
  Table[If[s ≤ n, 1, 0], {s, S}], n - 1]]
```

```
In[9]:= n = 150;
pmat = P[n]; // AbsoluteTiming
```

```
Out[10]= {37.0619, Null}
```

```
In[11]:= RV[l_, s_] := Module[{r = RandomInteger[pmat[[l + 1, s]] - 1], p = pmat[[l]]},
  Catch[
  Do[With[{w = p[[d]]}, If[r < w, Throw[s - d], r = r - w]], {d, Max[1, s - n], s - 1}]]]
```

```
In[12]:= Sample[] :=
  Reap[Module[{s = n (n + 1) / 2, v},
  Do[If[l == 0, Sow[s],
  v = RV[l, s];
  Sow[v];
  s = s - v],
  {l, n - 1, 0, -1}]]][[2, 1]]
```

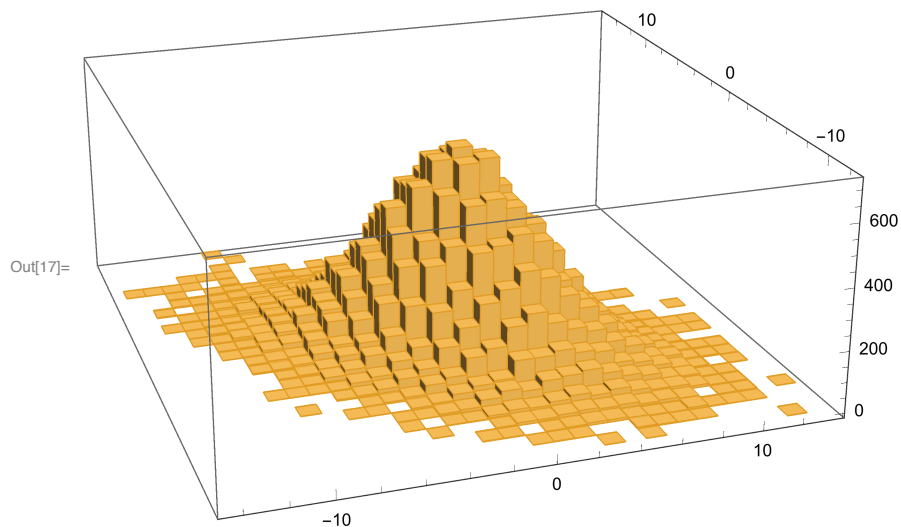
```
In[14]:= samples = ParallelTable[Sample[], {i, 40000}]; // AbsoluteTiming
```

```
Out[14]= {663.653, Null}
```

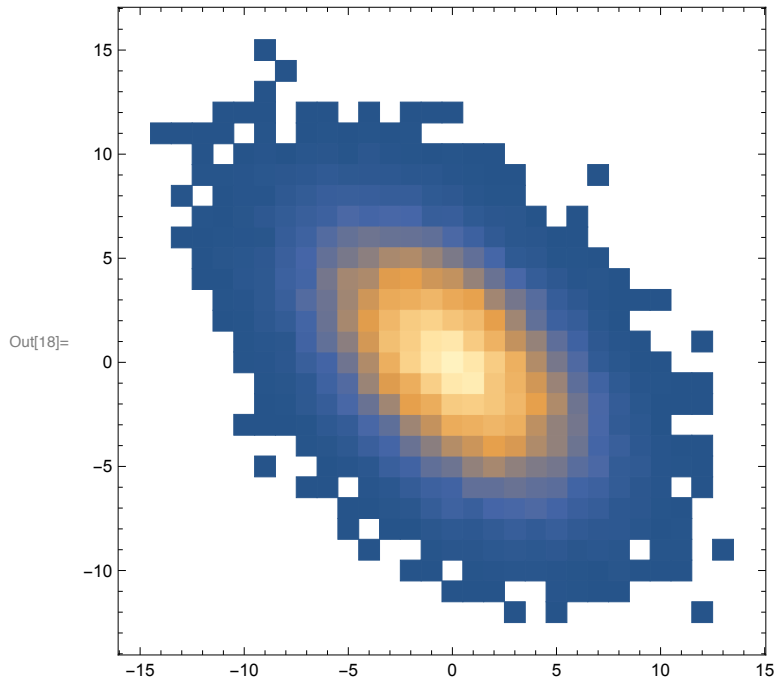
```
In[15]:= v[s_, j_] := Count[s, v_ /; v ≤ j] - j
```

```
In[16]:= points = Map[{v[#, n/3], v[#, 2 n/3]} &, samples];
```

```
In[17]:= Histogram3D[points, {1}]
```

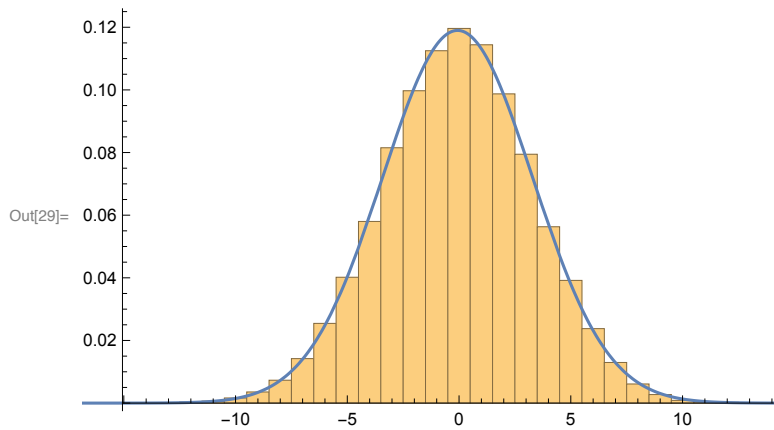


```
In[18]:= DensityHistogram[points, {1}]
```



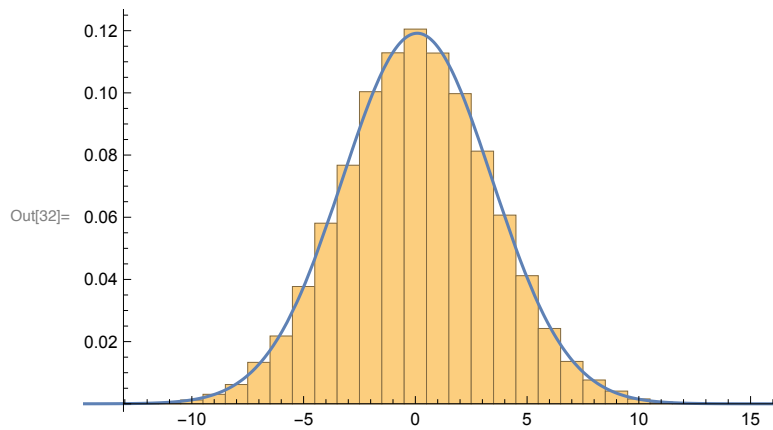
```
In[27]:= xpoints = Map[#[[1]] &, points];  
xfit = EstimatedDistribution[xpoints, NormalDistribution[mu, sigma]]  
Show[Histogram[xpoints, {1}, "PDF"],  
Plot[PDF[xfit, x], {x, -20, 20}, PlotRange -> All]]
```

Out[28]= NormalDistribution[-0.066325, 3.35274]



```
In[30]:= ypoints = Map[#[[2]] &, points];  
yfit = EstimatedDistribution[ypoints, NormalDistribution[mu, sigma]]  
Show[Histogram[ypoints, {1}, "PDF"],  
Plot[PDF[yfit, x], {x, -20, 20}, PlotRange -> All]]
```

```
Out[31]= NormalDistribution[0.08585, 3.34758]
```



```
In[33]:= fit = EstimatedDistribution[points,  
    MultinormalDistribution[{mux, muy}, {{sxx, sxy}, {syx, syy}}]]  
Show[Histogram3D[points, {1}, "PDF", ChartBaseStyle -> Opacity[0.5]], Plot3D[  
    PDF[fit, {x, y}], {x, -20, 20}, {y, -20, 20}, PlotRange -> All, PlotPoints -> 100]]  
Out[33]= MultinormalDistribution[{-0.066325, 0.08585},  
    {{11.2409, -5.65423}, {-5.65423, 11.2063}}]
```

