Integrable $\mathcal{E}$-models, 4d Chern-Simons theory & affine Gaudin-models.

based on [2008.01829] w/ M. Benini & A. Schenkel & [2011.13809] w/ S. Lacroix

2d Integrable field theories.

A field theory in 2d is classically integrable if its equations of motion take the Lax form:

$$\text{e.o.m.}\left(\{\phi_i\}\right) = 0 \iff \boxed{d\mathcal{L}(z) + \tfrac{1}{2}[\mathcal{L}(z), \mathcal{L}(z)] = 0}$$

Lax connection $\quad \mathcal{L}(z) = \underset{\uparrow}{\mathscr{L}(\sigma,\tau,z)}\, d\sigma + \underset{\uparrow}{\mathcal{M}(\sigma,\tau,z)}\, d\tau$

meromorphic dependence on z.

Examples:

1) Sinh - Gordon $\quad \partial_\sigma^2 \phi - \partial_\tau^2 \phi - m^2\left(e^{2\phi} - e^{-2\phi}\right) = 0$

$$\mathscr{L}(z) = \begin{pmatrix} \frac{1}{2}\partial_\tau \phi & \frac{m}{2}\left(z e^{\phi} + z^{-1} e^{-\phi}\right) \\ \frac{m}{2}\left(z e^{-\phi} + z^{-1} e^{\phi}\right) & -\frac{1}{2}\partial_\tau \phi \end{pmatrix}, \quad \mathcal{M}(z) = \begin{pmatrix} \frac{1}{2}\partial_\sigma \phi & -\frac{m}{2}\left(z e^{\phi} - z^{-1} e^{-\phi}\right) \\ -\frac{m}{2}\left(z e^{-\phi} - z^{-1} e^{\phi}\right) & -\frac{1}{2}\partial_\sigma \phi \end{pmatrix}$$

$$\hookrightarrow \quad d\mathcal{L}(z) + \tfrac{1}{2}[\mathcal{L}(z), \mathcal{L}(z)] \propto \begin{pmatrix} \text{e.o.m.} & 0 \\ 0 & \text{e.o.m.} \end{pmatrix} d\sigma \wedge d\tau.$$

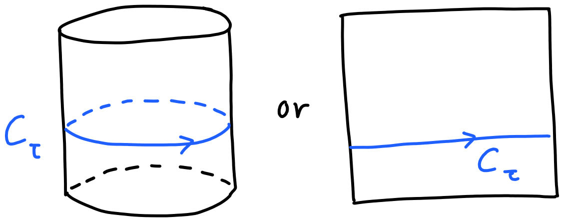
2) Principal chiral model $\quad \partial_+\big(\underbrace{g^{-1}\partial_- g}_{j_-}\big) - \partial_-\big(\underbrace{g^{-1}\partial_+ g}_{j_+}\big) = 0.$

Current $j := g^{-1} dg$

$$\mathcal{L}(z) = \frac{j - z * j}{1 - z^2} = \frac{j_+ \, d\sigma^+}{1 - z} + \frac{j_- \, d\sigma^-}{1 + z}$$

$$\hookrightarrow \quad d\mathcal{L}(z) + \tfrac{1}{2}[\mathcal{L}(z), \mathcal{L}(z)] = \frac{1}{1 - z^2}\left(dj + \tfrac{1}{2}[j, j]\right) - \frac{z}{1 - z^2} \, d * j .$$

Infinitely many integrals of motion:



$S^1 \times \mathbb{R}$ $\qquad$ $\mathbb{R}^2$

$$\partial_\tau \, \mathrm{tr}\left(P \overleftarrow{\exp} \int_{C_\tau} \mathcal{L}(z) \right) = 0$$

Q: What is the origin of the Lax connection?

Algebraic / Hamiltonian origin:

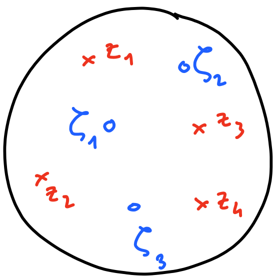
$\tilde{\mathfrak{g}} = \mathfrak{g} \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}k \oplus \mathbb{C}d$ $\qquad$ untwisted affine KM algebra.

dual bases $\{ I^{\tilde{a}} \} = \{ I^a \otimes t^n, k, d \}$, $\{ I_{\tilde{a}} \} = \{ I_a \otimes t^{-n}, d, k \}$

Idea: 2d IFTs with $\mathfrak{g}$-valued Lax connections as representations of Gaudin models associated with $\tilde{\mathfrak{g}}$:

[Feigin-Frenkel '07] [BV '17]

$$\sum_{i=1}^{N} \frac{\sum_{\tilde{\alpha}} I_{\tilde{\alpha}} \otimes I^{\tilde{\alpha}(i)}}{z - z_i} \, dz \xrightarrow{\text{rep.}} \omega\left(\partial_\sigma + \mathcal{L}(z,\sigma)\right)$$



$\mathbb{C}P^1 \setminus \{\zeta_j\}$

where $$\boxed{\omega = \sum_{i=1}^{N} \frac{k_i}{z - z_i} \, dz} = \frac{\prod_j (z - \zeta_j)}{\prod_i (z - z_i)} \, dz$$

$$\mathcal{L}(z,\sigma) = \sum_j \frac{\mathcal{J}_j(\sigma)}{z - \zeta_j}$$

Example: Principal chiral model

Gaudin Lax matrix

$$L(z) = \left(\frac{I_{\tilde{\alpha}} \otimes I^{\tilde{\alpha}(0)}_{[0]}}{z^1} + \frac{I_{\tilde{\alpha}} \otimes I^{\tilde{\alpha}(0)}_{[1]}}{z^2} - I_{\tilde{\alpha}} \otimes I^{\tilde{\alpha}(\infty)}_{[1]} \right) dz$$

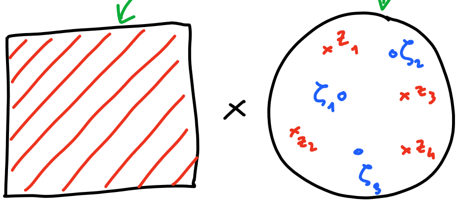
$$\xrightarrow{\text{rep.}} \underbrace{\left(\frac{1}{z^2} - 1\right) dz}_{\omega_{PCM}} \left(\partial_\sigma + \underbrace{\frac{1}{1 - z^2} \left(j_\sigma - z j_\tau \right)}_{\mathcal{L}_{PCM}(\sigma,\tau)} \right)$$

Geometric / Lagrangian origin:

Start from 4d Chern-Simons action:

[Costello '13]
[Costello-Witten-Yamazaki '17, '18]
[Costello-Yamazaki '19]

$$S_\omega(A) = \frac{i}{4\pi} \int_{\Sigma \times C =: X} \omega \wedge CS(A)$$



$\mathbb{R}^2 \times \mathbb{C}P^1 \setminus \{\zeta_j\}$

- $\omega = \sum_{i=1}^{N} \frac{k_i}{z - z_i} \, dz$
- $A = A_\sigma d\sigma + A_\tau d\tau + A_{\bar{z}} d\bar{z} \in \Omega^1(X, \mathfrak{g})$
- $CS(A) = \left\langle A, dA + \frac{1}{3}[A, A] \right\rangle$

Remark: ω is singular on surface defects:

$(\underline{z} = \{\text{poles of } \omega\})$ $\qquad D = \bigsqcup_{x \in \underline{z}} \Sigma_x \,, \qquad \Sigma_x := \Sigma \times \{x\}.$

Nevertheless, $\omega \wedge CS(A)$ is locally integrable near D.

Behaviour of $S_\omega(A)$ under (finite) gauge transformations

$$A \xmapsto{g \in C^\infty(X,G)} {}^gA := -dg\,g^{-1} + g A g^{-1} \quad ?$$

$$S_\omega({}^gA) = S_\omega(A) + \frac{i}{4\pi}\int_X \omega \wedge d\langle g^{-1}dg, A\rangle + \frac{i}{24\pi}\int_X \omega \wedge \langle g^{-1}dg, [g^{-1}dg, g^{-1}dg]\rangle$$

[Remark: For usual 3d CS, $S_{3d}(A) = \frac{k}{4\pi}\int_{M_3} CS(A)$ $\quad (k \in \mathbb{Z})$

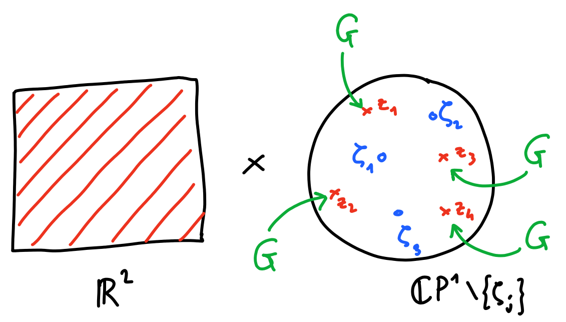
$$S_{3d}({}^gA) = S_{3d}(A) + 2\pi k N \,, \quad \text{for some } N \in \mathbb{Z} \;]$$

Define defect (Lie) group

$$G^{\underline{z}} := \prod_{x \in \underline{z}} G$$

and defect (Lie) algebra

$$\mathfrak{g}^{\underline{z}} := \prod_{x \in \underline{z}} \mathfrak{g}$$



with bilinear form $\langle\!\langle \cdot , \cdot \rangle\!\rangle_\omega : \mathfrak{g}^{\mathfrak{z}} \times \mathfrak{g}^{\mathfrak{z}} \longrightarrow \mathbb{C}$:

$$\langle\!\langle X, Y \rangle\!\rangle_\omega = \sum_{x \in \mathfrak{z}} k_x \langle X_x , Y_x \rangle \qquad X = (X_x), Y = (Y_x) \in G^{\mathfrak{z}}.$$

Consider embedding

$$\iota : D = \bigsqcup_{x \in \mathfrak{z}} \Sigma_x \hookrightarrow X .$$

<u>Note:</u> Pullback of $g \in C^\infty(X, G)$ & $A \in \Omega^1(X, \mathfrak{g})$ are:

- $\iota^* g \in C^\infty(D, G) = C^\infty\big(\bigsqcup_{x \in \mathfrak{z}} \Sigma_x , G\big)$

$$\cong \prod_{x \in \mathfrak{z}} C^\infty(\Sigma_x, G) \cong C^\infty(\Sigma, G^{\mathfrak{z}})$$

- $\iota^* A \in \Omega^1(D, \mathfrak{g}) \cong \Omega^1(\Sigma, \mathfrak{g}^{\mathfrak{z}})$.

$$S_\omega({}^g A) = S_\omega(A) - \frac{1}{2} \int_\Sigma \langle\!\langle (\iota^* g)^{-1} d_\Sigma (\iota^* g), \iota^* A \rangle\!\rangle_\omega$$

[Localised on or "near" defect D.]

$$+ \frac{1}{12} \int_{\Sigma \times [0,1]} \langle\!\langle \hat{g}^{-1} d\hat{g}, [\hat{g}^{-1} d\hat{g}, \hat{g}^{-1} d\hat{g}] \rangle\!\rangle_\omega$$

$\hat{g} \in C^\infty(\Sigma \times [0,1], G^{\mathfrak{z}})$

s.t. $\hat{g}|_0 = \iota^* g , \ \hat{g}|_1 = e$.

<u>Two ways of ensuring gauge invariance:</u>

Let $k \subset \mathfrak{g}^{\mathfrak{z}}$ be an isotropic Lie subalgebra.

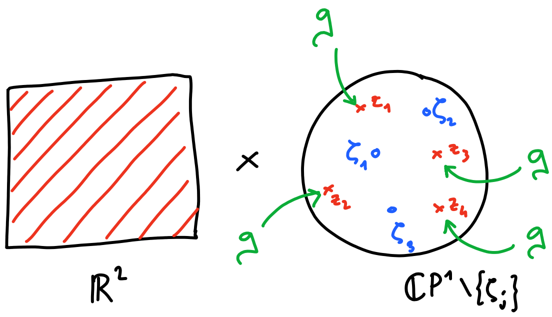
$$\langle\!\langle X, Y \rangle\!\rangle_\omega = 0 \qquad \forall X, Y \in k.$$

Let $K \subset G^{\mathfrak{z}}$ corresponding connected Lie subgroup.

1) Strict boundary conditions

Consider bulk fields $A \in \Omega^1(X, \mathfrak{g})$ and gauge transformations $g \in C^\infty(X, G)$ satisfying

- $\iota^* A \in \Omega^1(\Sigma, \mathfrak{k}) \subset \Omega^1(\Sigma, \mathfrak{g}^{\underline{z}})$
- $\iota^* g \in C^\infty(\Sigma, K) \subset C^\infty(\Sigma, G^{\underline{z}})$



$\iota^* A = (A|_x)_{x \in \underline{z}} \in \Omega^1(\Sigma, \mathfrak{k})$

$A|_x \in \Omega^1(\Sigma, \mathfrak{g})$

"non-local" boundary condition on $\mathbb{C}P^1$.

Theorem (Benini-Schenkel-BV)

$S_\omega(A)$ is gauge invariant.

Summary: Strict boundary conditions imposed via a pullback construction:

$\iota^* A = B$

$\iota^* g = k$

$$
\begin{tikzcd}
\left\{ \begin{array}{c} \text{Bulk fields and} \\ \text{gauge transformations} \\ \text{satisfying b.c.} \end{array} \right\} \arrow[r, dashed] \arrow[d, dashed] & \underset{\text{Bulk fields}}{\left\{ \begin{array}{l} \cdot A \in \Omega^1(X, \mathfrak{g}) \\ \cdot g \in C^\infty(X, G) \end{array} \right\}} \arrow[d, "\iota^*"] \\
\left\{ \begin{array}{l} \cdot B \in \Omega^1(\Sigma, \mathfrak{k}) \\ \cdot k \in C^\infty(\Sigma, K) \end{array} \right\} \arrow[r, hook] & \left\{ \begin{array}{l} \cdot \Omega^1(\Sigma, \mathfrak{g}^{\underline{z}}) \\ \cdot C^\infty(\Sigma, G^{\underline{z}}) \end{array} \right\}
\end{tikzcd}
$$

"boundary" fields

2) Homotopical boundary conditions

Gauge fields are not compared by equality but rather via gauge transformations.

$\rightsquigarrow$ Impose boundary conditions via homotopy pullback (i.e. impose them up to gauge transformations):

$$\begin{tikzcd}
\text{homotopy pullback} \arrow[r, dashed] \arrow[d, dashed] \arrow[dr, phantom, "h"] & \overset{\text{Bulk fields}}{\left\{\begin{array}{l}\bullet A \in \Omega^1(X,\mathfrak{g}) \\ \bullet g \in C^\infty(X,G)\end{array}\right\}} \arrow[d, "\iota^*"] \\
\underset{\text{"boundary" fields}}{\left\{\begin{array}{l}\bullet B \in \Omega^1(\Sigma,\mathfrak{k}) \\ \bullet k \in C^\infty(\Sigma,K)\end{array}\right\}} \arrow[r, hook] & \left\{\begin{array}{l}\bullet\ \Omega^1(\Sigma,\mathfrak{g}^\Sigma) \\ \bullet\ C^\infty(\Sigma,G^\Sigma)\end{array}\right\}
\end{tikzcd}$$

A model for the homotopy pullback:

- Fields:
$$\left.\begin{array}{l} A \in \Omega^1(X,\mathfrak{g}) \\ h \in C^\infty(\Sigma,\mathfrak{g}^\Sigma) \end{array}\right\} \quad {}^h(\iota^* A) = B \in \Omega^1(\Sigma,\mathfrak{k})$$
edge mode

- Gauge transformations: $(g \in C^\infty(X,G),\ k \in C^\infty(\Sigma,K))$
$$A \longmapsto {}^gA = -dg\, g^{-1} + g A g^{-1}$$
$$h \longmapsto k\, h\, (\iota^* g)^{-1}$$

Couple edge mode to gauge field:

$$S^{\text{ext}}_\omega(A,h) := S_\omega(A) - \frac{1}{2}\int_\Sigma \langle\!\langle h^{-1} d_\Sigma h,\ \iota^* A\rangle\!\rangle_\omega$$

$$+\frac{1}{12}\int_{\Sigma\times[0,1]} \langle\!\langle \hat{h}^{-1}d\hat{h}, [\hat{h}^{-1}d\hat{h}, \hat{h}^{-1}d\hat{h}] \rangle\!\rangle_\omega$$

$\hat{h} \in C^\infty(\Sigma \times [0,1])$
s.t. $\hat{h}|_0 = h$, $\hat{h}|_1 = e$.

Theorem (Benini - Schenkel - BV)

Extended action $S^{ext}_\omega(A,h)$ is gauge invariant.

From 4d CS + edge mode to 2d IFTs

Want to turn 4d CS gauge field

$$A = A_\sigma(\sigma,\tau,z)d\sigma + A_\tau(\sigma,\tau,z)d\tau + A_{\bar{z}}(\sigma,\tau,z)d\bar{z}$$

into the 2d IFT Lax connection

$$\mathcal{L}(z) = \mathcal{L}(\sigma,\tau,z)d\sigma + \mathcal{M}(\sigma,\tau,z)d\tau$$

- Step 1: Restrict attention to

$$\mathcal{L} := A = A_\sigma(\sigma,\tau,z)\,d\sigma + A_\tau(\sigma,\tau,z)\,d\tau \in \Omega^{1,0,0}(X,\mathfrak{g})$$

and $g \in C^\infty(X, G)$ such that $\bar{\partial} g g^{-1} = 0$.

Extended action now reads:

$$S^{ext}_\omega(\mathcal{L},h) = \frac{i}{4\pi}\int_X \omega \wedge \langle \mathcal{L}, \bar{\partial}\mathcal{L}\rangle - \frac{1}{2}\int_\Sigma \langle\!\langle h^{-1}d_\Sigma h, \iota^*\mathcal{L}\rangle\!\rangle_\omega$$

$\left[{}^h(\iota^*\mathcal{L}) \in \Omega^1(\Sigma,k)\right]$

$$+\frac{1}{12}\int_{\Sigma\times[0,1]} \langle\!\langle \hat{h}^{-1}d\hat{h}, [\hat{h}^{-1}d\hat{h}, \hat{h}^{-1}d\hat{h}] \rangle\!\rangle_\omega$$

Equations of motion:

* <u>Bulk e.o.m.</u>: $\bar{\partial}\mathcal{L} = 0$ on $X = \Sigma \times (\mathbb{C}P^1 \setminus \{\zeta_j\})$

* <u>Defect e.o.m.</u>: $d_\Sigma(\iota^*\mathcal{L}) + \frac{1}{2}[\iota^*\mathcal{L}, \iota^*\mathcal{L}] = 0$ on Σ.

[<u>N.B.</u>: Flatness of $\iota^*\mathcal{L} \in \Omega^1(\Sigma, G^{\frac{2}{-}})$ and <u>not</u> of $\mathcal{L} \in \Omega^1(X, G)$.]

- <u>Step 2</u>: Restrict attention to <u>particular solutions of bulk e.o.m.</u>:

Call $\mathcal{L} \in \Omega^{1,0,0}(X, \mathfrak{g})$ admissible if it is:

(a) meromorphic on $\mathbb{C}P^1$ with poles at $\underline{\zeta}$ (zeroes of ω)

(b) such that $\omega \wedge \mathcal{L}$ is bounded near $\underline{\zeta}$,

(c) and $\omega \wedge (d_\Sigma \mathcal{L} + \frac{1}{2}[\mathcal{L}, \mathcal{L}])$ is bounded near $\underline{\zeta}$.

<u>Proposition</u>: If $\mathcal{L} \in \Omega^{1,0,0}(X, \mathfrak{g})$ is admissible then

$$\underbrace{d_\Sigma(\iota^*\mathcal{L}) + \frac{1}{2}[\iota^*\mathcal{L}, \iota^*\mathcal{L}] = 0 \text{ on } \Sigma}_{\text{(defect e.o.m.)}} \iff d_\Sigma \mathcal{L} + \frac{1}{2}[\mathcal{L}, \mathcal{L}] = 0 \text{ on } X.$$

Extended action now reads:

$$S^{ext}_\omega(\mathcal{L}, h) = -\frac{1}{2}\int_\Sigma \langle\!\langle h^{-1} d_\Sigma h, \iota^*\mathcal{L} \rangle\!\rangle_\omega + \frac{1}{12}\int_{\Sigma\times[0,1]} \langle\!\langle \hat{h}^{-1} d\hat{h}, [\hat{h}^{-1} d\hat{h}, \hat{h}^{-1} d\hat{h}] \rangle\!\rangle_\omega$$

$[{}^h(\iota^*\mathcal{L}) \in \Omega^1(\Sigma, k)]$

and equations of motion now read:

$$\boxed{d_\Sigma \mathcal{L} + \frac{1}{2}[\mathcal{L}, \mathcal{L}] = 0} \quad \text{on } X.$$

- <u>Step 3</u>: Solve the constraint

$${}^h(\iota^*\mathcal{L}) \in \Omega^1(\Sigma, k)$$

for admissible $\mathcal{L}$ in terms of $h \in C^\infty(\Sigma, G^{\underline{z}})$.

We assume ω has a double pole at ∞, and partially fix gauge invariance to fix components of $h \in C^\infty(\Sigma, G^{\underline{z}})$ at ∞: $\quad (\underline{z}' := \underline{z} \setminus \{\infty\})$

$$h \in C^\infty(\Sigma, G^{\underline{z}}) \rightsquigarrow \ell \in C^\infty(\Sigma, G^{\underline{z}'})$$

$$\iota : D = \bigsqcup_{x \in \underline{z}} \Sigma_x \hookrightarrow X \quad \rightsquigarrow \quad j : D = \bigsqcup_{x \in \underline{z}'} \Sigma_x \hookrightarrow X$$

$${}^h(\iota^*\mathcal{L}) \in \Omega^1(\Sigma, k) \rightsquigarrow {}^\ell(j^*\mathcal{L}) \in \Omega^1(\Sigma, k').$$

To satisfy admissibility conditions (a) & (b), take:

$$\mathcal{L} = \sum_{y \in \underline{\zeta}} \frac{\mathcal{L}_y}{z - y}$$

- $\underline{\zeta}$ set of zeroes of ω
- $\mathcal{L}_y \in \Omega^1(\Sigma, \mathfrak{g})$

To satisfy admissibility condition (c), require

$$\mathcal{E}(j^*\mathcal{L}) = *(j^*\mathcal{L}) \qquad \text{[Ševera '16]}$$

where $\mathcal{E} : \mathfrak{g}^{\underline{z}'} \xrightarrow{\cong} \mathfrak{g}^{\underline{z}'}$ is defined by $(\underline{\zeta} = \underline{\zeta}^+ \sqcup \underline{\zeta}^-)$

$$\begin{array}{ccc}
\mathfrak{g}^{\underline{z}'} \ni X & \xleftarrow[j^*]{\cong\ (|\underline{\zeta}| = |\underline{z}'|)} & \displaystyle\sum_{y \in \underline{\zeta}^+} \frac{f_y}{z - y} + \sum_{y \in \underline{\zeta}^-} \frac{f_y}{z - y} \\
\Big\downarrow \mathcal{E} & & \Big\downarrow \\
\mathfrak{g}^{\underline{z}'} \ni \mathcal{E}X & \xleftarrow[j^*]{\cong} & \displaystyle\sum_{y \in \underline{\zeta}^+} \frac{f_y}{z - y} - \sum_{y \in \underline{\zeta}^-} \frac{f_y}{z - y}
\end{array}$$

We have (under mild assumptions)

$$\mathfrak{g}^{\mathbf{z}'} = \mathrm{Ad}_\ell^{-1} \mathfrak{k} \oplus \mathcal{E}\, \mathrm{Ad}_\ell^{-1} \mathfrak{k} \qquad \forall \ell \in G^{\mathbf{z}'}.$$

Define $P_\ell : \mathfrak{g}^{\mathbf{z}'} \longrightarrow \mathcal{E}\,\mathrm{Ad}_\ell^{-1}\mathfrak{k}$ projector with $\ker P_\ell = \mathrm{Ad}_\ell^{-1}\mathfrak{k}$.

Proposition. (Lacroix - BV) There is a unique solution of the constraint ${}^{\ell}(j^*\mathcal{L}) \in \Omega^1(\Sigma, \mathfrak{k}')$ such that $\mathcal{E}(j^*\mathcal{L}) = *(j^*\mathcal{L})$, given by

$$j^*\mathcal{L} = (1 - P_\ell)\,\mathcal{E}(\ell^{-1} * d_\Sigma \ell) + P_\ell(\ell^{-1} d_\Sigma \ell).$$

Resulting 2d integrable σ-model on $K' \backslash G^{\mathbf{z}'}$,

$$S_\omega^{ext}(\ell) = -\frac{1}{2}\int_\Sigma \langle\!\langle \ell^{-1} d_\Sigma \ell, (1-P_\ell)\,\mathcal{E}(\ell^{-1} * d_\Sigma \ell) + P_\ell(\ell^{-1} d_\Sigma \ell) \rangle\!\rangle_\omega$$
$$+ \frac{1}{12}\int_{\Sigma \times [0,1]} \langle\!\langle \hat{\ell}^{-1} d\hat{\ell}, [\hat{\ell}^{-1} d\hat{\ell}, \hat{\ell}^{-1} d\hat{\ell}] \rangle\!\rangle_\omega$$

known as an $\mathcal{E}$-model. [Klimčík - Ševera '95]

$$\begin{tikzcd}
 & \mathcal{E}\text{-model on } G^{\mathbf{z}'} \arrow[dl] \arrow[dr] & \\
\sigma\text{-model on } K' \backslash G^{\mathbf{z}'} \arrow[rr, dashed, leftrightarrow, "\text{P-L}", "\text{T-duality}"'] & & \sigma\text{-model on } \tilde{K}' \backslash G^{\mathbf{z}'}
\end{tikzcd}$$

Conclusions:

- Passage from 4d to 2d via edge modes:

4d CS
w/edge modes

4d CS w/ b.c. $\nearrow$ $\nwarrow$ 2d σ-model ($\mathcal{E}$-model)

* Edge modes via homotopy pull back.
[Mathieu - Murray - Schenkel - Teh '20]

* Generalises to higher order poles in ω.

* More general solution to constraint?

- Hamitonian analysis?
  * Hamiltonian integrability (r-matrix, ...)
  * Affine Gaudin model description? [BV'19]

$$\left\{\begin{matrix}\text{boundary conditions} \\ \text{in 4d CS theory}\end{matrix}\right\} \overset{?}{\longleftrightarrow} \left\{\begin{matrix}\text{representations} \\ \text{of } \tilde{\mathfrak{g}}\end{matrix}\right\}$$

- Quantization
  * Relationship between quantum 4d CS & 2d QIFT ?
  * Quantum 4d CS vs. affine quantum Gaudin model ?
  * ...