

Vertex operators and screening charges as gauge-invariant brane intersections in twisted M-theory

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An analogy: B-model

- IIB on $CY \times \mathbb{R}^4$ + “self-dual graviphoton background”
 - Protected subsector: B-model aka Kodaira-Spencer (BCOV) on CY
 - 6d “holomorphic gravity”, partition function computes F-terms in 4d
- Modern perspective: Omega-deformed twisted supergravity on $CY \times \mathbb{C}_\epsilon \times \mathbb{C}_{-\epsilon}$
- Branes on $M \times \mathbb{C}_\epsilon$: Omega-deformed world-volume theory (hCS, etc.)
 - KS theory non-renormalizable, but unique counterterms with hCS

Twisted M-theory

- Twisted M-theory on $[\mathbb{R} \times \mathbb{C} \times \mathbb{C}] \times [\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{C}_{-\epsilon_1-\epsilon_2}]$
 - Special case of $\mathbb{R} \times M_4 \times CY_\epsilon$ or $M_4 \times G_{2;\epsilon}$
 - 5d theory of complex symplectic-topological gravity
- Three dual descriptions as hol-top 5d non-commutative U(1) CS theory
 - Non-renormalization theorems
 - M5 branes? M2 branes? Brane intersections?

Motivations

- **Holomorphic twists:** SUSY unexplored frontier?
- **Twisted SUGRA:** novel non-renormalization theorems? Holography?
- **Vertex Operator Algebras and SUSY gauge theory:** AGT and beyond

Twisting SUSY

- Twist:
 - Pick nilpotent supercharge Q
 - Use Q as BRST charge
- Topological twist: all translations are Q -exact.
- Holomorphic twist: all anti-holomorphic translations are Q -exact.
- Dependence on couplings or RG scale may be controllable or unique.

Low SUSY examples

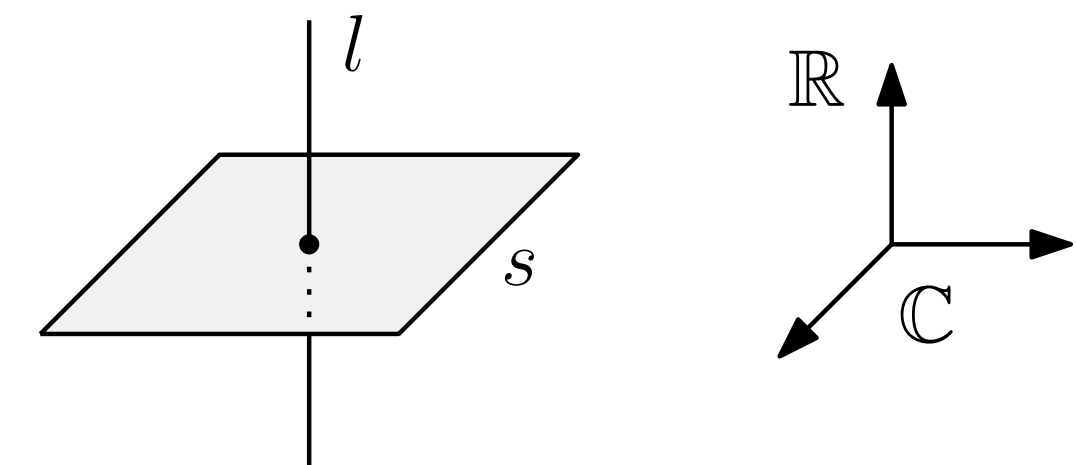
- 3d $N=2$ admit holomorphic-topological twist ($R \times C$)
- 4d $N=1$ admit holomorphic twist ($C \times C$)
- 5d $N=1$ admit holomorphic-topological twist ($R \times C \times C$)
- Interesting physics! Anyons, confinement, UV fixed points. What survives twist?
- For SCFT, involves operators in superconformal index. Fun holography, even black holes.

Twisted SUGRA

- Twisted SUGRA: turn on nilpotent vev for super-ghost
 - Automatically twists SQFT coupled to it
 - Simplified theory often has surprising non-renormalization theorems.
- Sometimes Omega deformation is possible
 - Localize to isometry fixed points, lowers dimensionality.

Reducing to 5d

- M-theory on CY manifold \rightarrow 5d N=1 SUSY
- Twisted M-theory on CY manifold \rightarrow Holomorphic-topological twist of 5d
 - Local $\mathbb{R} \times \mathbb{C}^2$ geometry, complex symplectic
 - M2 branes on curve in CY \rightarrow topological line defects along \mathbb{R}
 - M5 branes on surface in CY \rightarrow holomorphic surface defects along \mathbb{C}
- Little known for general 5d theory or general CY.



Non-commutative 5d CS theory

- Costello: twisted M-theory on $\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2} \times \mathbb{C}_{-\epsilon_1-\epsilon_2}$ has dual gauge theory description(s)

$$\frac{1}{\epsilon_1} \int \left[A \wedge_{*\epsilon_2} dA + \frac{2}{3} A \wedge_{*\epsilon_2} A \wedge_{*\epsilon_2} A \right] dz_1 dz_2$$

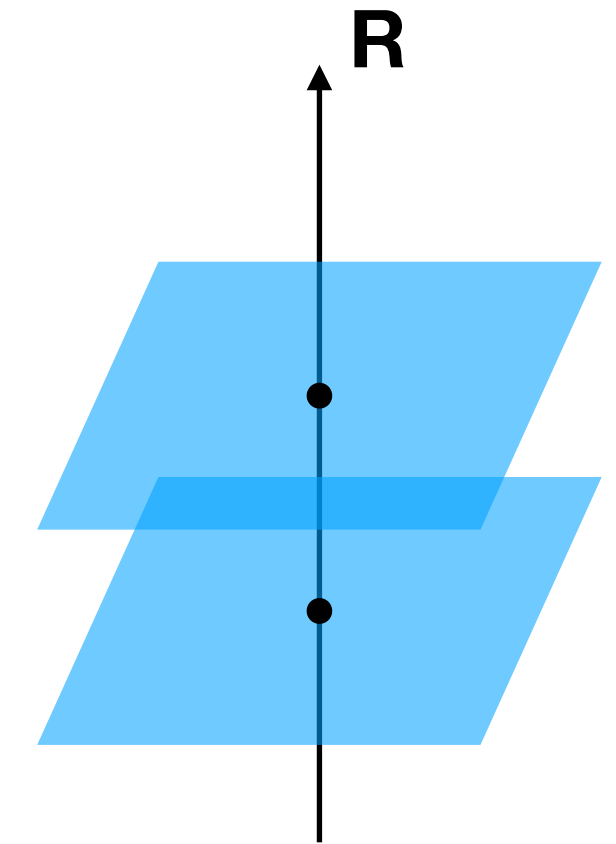
- Permutation symmetry hidden.
 - M2 map to Wilson lines of charge 1, or Wilson lines of charge -1, or instanton particles. Weyl algebra $[z_1, z_2] = \epsilon_i$
 - M5 map to 't Hooft surfaces of charge 1, or 't Hooft surfaces of charge -1, or coupling to chiral fermion. Free boson VOA $u(1)_{-\frac{1}{\epsilon_i \epsilon_j}}$

M5 defects

- N M5 branes support 6d SCFT, coupled to 11d SUGRA
- Twist: 6d SCFT on $\mathbb{C}_{\epsilon_1} \times \mathbb{C}_{\epsilon_2}$ gives 2d chiral algebra W_N .
 - Origin of AGT
- Coupling to twisted SUGRA?
 - Koszul duality: VOA map $W_\infty \rightarrow V$ required to couple V to 5d bulk
- Three stacks of M5 give “corner VOA” Y_{LMN}

Fusion and coproducts

- Can we fuse N separate M5s into a single stack?
 - Coproduct $W_\infty \rightarrow W_\infty \otimes W_\infty$ governs fusion of defects.
- Single M5 is a free boson. Free field realization of W_N from N bosons
 - A piece of Coulomb gas construction.
 - Can we find screening charges, etc.? Yes! M2's stretched between M5's

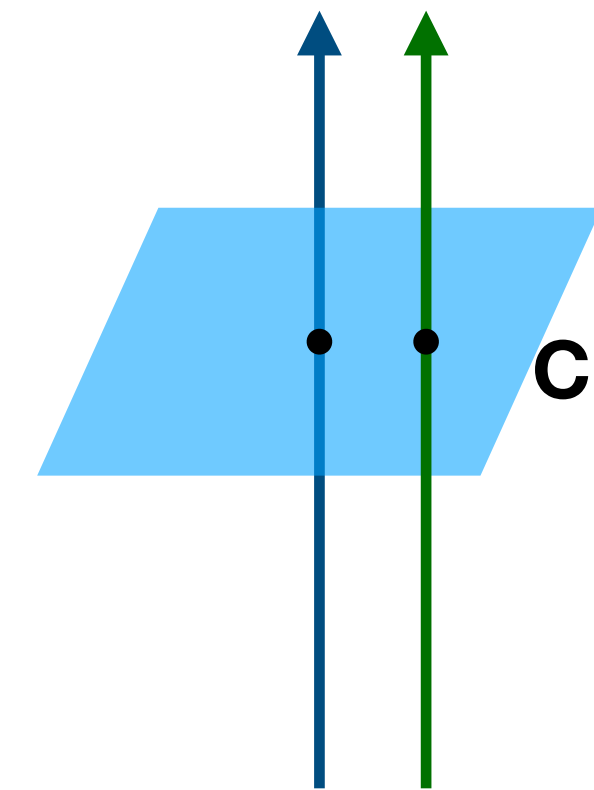


M2 defects

- N M2 branes support 3d SCFT, coupled to 11d gravity
- Twist: 3d SCFT on $\mathbb{R} \times \mathbb{C}_{\epsilon_1}$ gives 1d algebra SHN
 - Generated by Calogero Hamiltonian and symmetric polynomials
- Coupling to twisted SUGRA?
 - Koszul duality: algebra map $A \rightarrow B$ require to couple B to 5d bulk
- Three stacks of M2 give which algebra?

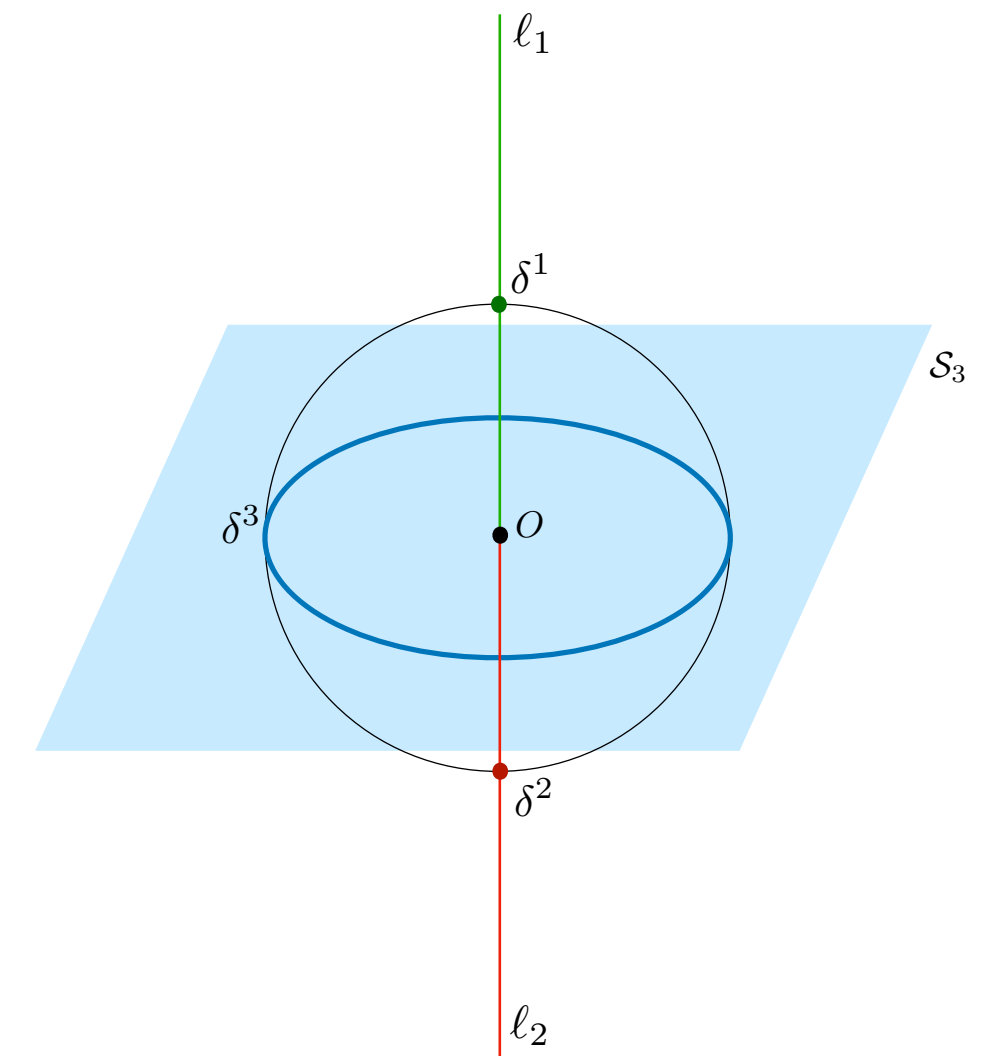
Fusion and coproducts

- Can we fuse N separate M2s into a single stack?
 - Meromorphic coproduct $A \rightarrow A \otimes A[[z]]$ governs fusion.
- Single M2 is a Weyl algebra.
 - Coproduct gives Calogero-like presentation for three-stack defect algebra. Novel integrable system.



Junctions

- AGT: M2 ending on M5 give degenerate vertex operators.
- BPS M2-M5 intersections can be coupled to supergravity.
- Twist: gauge invariance controlled by a mixed coproduct
 - Junction must intertwine two A actions: local operators from bottom half-line vs coproduct $A \rightarrow A \otimes W_\infty$ of local operators from top half-line and VOA modes from surface



Gauge-invariant junctions

- A single M2 ending on fundamental degenerate field from below
- A single M2 ending on anti-fundamental degenerate field from above
- A single M2 crossing an M5 at a Miura operator
- Fusion of the above in topological or holomorphic directions
- M-theory interpretation for Miura transform, Coulomb gas constructions, VOA construction of Calogero wavefunctions and more.

To Do

- M5s with different orientation in $C \times C$
- q -deformation from $R \times C^* \times C^*$
- Changing inner toric CY
 - No faces: matrix generalizations of VOA and Calogero, etc.
 - Faces: unknown. Holomorphic-topological twist of 5d gauge theories?
- Interfaces: topological twist of 4d $N=1$ domain walls