Stable Envelopes and Janus

A QFT Home For Some Geometric constructions

[Based on w.i.p. with N. Nekrasov]

I. Introduction

→ Study physical realizations of a few constructions in mathematics, namely by:

(a) Maulik-Okounkov, Aganagic-Okounkov, Okounkov-Smirnov, ... : Stable Envelopes, R-matrices

(b) Nakajima, Varagnolo : $K_T(X)$, $H_T(X)$, $Ell_T(X)$

Both (a) & (b) constructions look very "physical."
Maps between $H_T(X_1)$ and $H_T(X_2)$ defined via correspondences remind interfaces ... between QFT_1 and QFT_2

$QFT_1 \leftrightarrow X_1$, $QFT_2 \leftrightarrow X_2$

In fact, this **can** be viewed as part of Bethe/Gauge correspondence [Nekrasov-Shatashvili]

→ Main idea: SUSY Interfaces between SUSY QFTs
$\Longrightarrow$ linear maps between SUSY vacua
⟶ sought after structures.

E.g. $QFT_1 \mid QFT_2 \mid QFT_3 \longrightarrow QFT_1 \mid QFT_3$

time ⟶   time ⟶

→ Study theories w/ 8 supercharges:

1d $\mathcal{N}=8$ on $\mathbb{R}$	2d $\mathcal{N}=(4,4)$ on $S^1\times\mathbb{R}$	3d $\mathcal{N}=4$ on $E_\tau\times\mathbb{R}$
$\Downarrow$	$\Downarrow$	$\Downarrow$
H_T, $Y_\hbar$	K_T, $L_\hbar$	Ell_T, $E_\hbar$

- Will identify SUSY Janus interfaces — building blocks (spoiler: stable envelopes)
- Compute their matrix elements.
- Phase transitions = stable envelops ; Wall-crossing = R- matrices

II. 3d $\mathcal{N}=2$, $\mathcal{N}=4$ theories, quivers, etc...

- 3d $\mathcal{N}=2$ theory is determined by: $\left\{ \begin{array}{l} \text{Gauge group } G \text{ ; its } \mathbb{C}\text{-rep. } R \text{ ;} \\ \text{Superpotential } W \in \mathbb{C}[R]^W \\ \text{real masses } m_{\mathbb{R}} \text{ ; real F.I. terms } \zeta_{\mathbb{R}} \end{array} \right\}$

  ↑ GLSM

- Focus on $\mathcal{N}=4$ theories with matter in $\mathbb{H}$-rep $R \oplus \overline{R}$, where R – $\mathbb{C}$-rep.

  Viewed as an $\mathcal{N}=2$ theory, it has matter (chiral multiplets) $(Q, \widetilde{Q}, \varphi)$

  where $Q \in R$ $\quad \widetilde{Q} \in \overline{R}$ $\quad \varphi \in \mathrm{adj}$, $\varphi = \sigma_{\mathbb{C}}$

  And $W = \mathrm{Tr}\, \widetilde{Q}\varphi Q + \mathrm{Tr}\, \widetilde{Q}\, m_{\mathbb{C}}\, Q + \mathrm{Tr}\, \zeta_{\mathbb{C}}\varphi$

  $\mathcal{N}=2$ R-symmetry $U(1)_r = \mathrm{Diag}(U(1)_H \times U(1)_C)$, $U(1)_H \times U(1)_C \subset SU(2)_H \times SU(2)_C$

  $\mathcal{N}=2$ flavor symmetry is $G_F^{\mathcal{N}=2} = G_F^{\mathcal{N}=4} \times U(1)_\hbar$, $U(1)_\hbar = \mathrm{Adiag}(U(1)_H \times U(1)_C)$

  Masses and flat connections for $U(1)_\hbar$ "softly break" $\mathcal{N}=4$ to $\mathcal{N}=2$

  → 2d and 1d analogs are obtained by dimensional reduction.

We study 3d on $E_\tau \times \mathbb{R}$; 2d on $S^1 \times \mathbb{R}$; 1d on $\mathbb{R}$

↑ Elliptic ; ↑ Trigonometric ; ↑ Rational

Notation: X_H = Higgs branch, X_C = 3d Coulomb branch.

Think of quiver theories, so X_H = quiver variety.

We work equivariantly w.r.t. $\mathbb{T} \equiv \mathbb{T}_F^{\mathcal{N}=2} = \mathbb{T}_F^{\mathcal{N}=4} \times U(1)_\hbar$ – max. torus of flavor gp.

Interested in $H_{\mathbb{T}}(X_H)$, $K_{\mathbb{T}}(X_H)$, $Ell_{\mathbb{T}}(X_H)$

→ Must preserve $U(1)_\hbar$ (so no complex masses $m_{\mathbb{C}}$ or complex FI $\zeta_{\mathbb{C}}$!)

→ Can have real masses $m_{\mathbb{R}} \in \underbrace{\mathrm{Lie}(\mathbb{T}_F^{\mathcal{N}=4})}_{\text{chambers}}$ and real FI $\zeta_{\mathbb{R}} \in \underbrace{\mathrm{Lie}(\mathbb{T}^{top})}_{\text{chambers}}$

resolve X_C ; resolve X_H

III. Supercharges and constructions in Q-cohomology

Consider our theory on $E_\tau \times \mathbb{R}$, $y \in \mathbb{R}$ – Euclidean time, $\mathcal{H}[E_\tau]$-Hilbert space

Pick $Q \in$ SUSY algebra. It acts both on $\mathcal{H}[E_\tau]$ and on operators.

Classic path to interesting math: pass to Q-cohomology (possibly equivariant).

Standard argument: W.r.t. a Hermitian $\langle , \rangle$ on $\mathcal{H}[\Sigma]$, there is $Q^\dagger$,

$\Rightarrow$ Q-cohomology on $\mathcal{H}[E_\tau] \simeq \ker\{Q, Q^\dagger\}$ (like in Hodge theory)

Often $\{Q, Q^\dagger\} = H$ is a Hamiltonian $\Rightarrow$ $\ker H$ = SUSY ground states (vacua)

Q-closed observables lead to linear operators on the space of vacua.

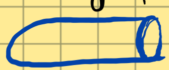
Interesting supercharges in 3d $\mathcal{N}=2$

- Q_A = 3d version ("uplift") of the 2d A-model supercharge (Gromov-Witten theory, Fukaya-Seidel cat....)

  lift of A-branes = (1,1) boundary conditions

  $Q_A^2 = i\partial_\varphi + g(\sigma_{\mathbb{R}}, m_{\mathbb{R}})$ "3d A-twist" (Benini-Zaffaroni; Closset-Kim-Willet)

Interesting partition function

on  $\times_q S^1 \Rightarrow$ K-theoretic count of quasimaps

$\quad$ [in 2d - cohomological]

$\quad$ ‖

"Vertex function"

- Q_B = 3d uplift of the B-model supercharge. B-branes = (0,2) boundary conditions

  "3d holomorphic-topological twist" (Costello-Dimofte-Gaiotto)

- $Q = Q_A + Q_A^\dagger = Q_B + Q_B^\dagger$ = 3d uplift of the Ω-deformation supercharg

  "OUR" supercharge; common Q preserved by A and B branes. $\rightarrow$ 3d $\mathcal{N}=1$ supercharge

  Consider theory on $E_\tau \times \mathbb{R}_y$

  $Q^2 = D_{\bar{z}}$ $\quad$ z - complex coordinate on E_τ.

  - Q-cohomology on $\mathcal{H}[E_\tau] \simeq$ "SUSY vacua on E_τ"
  - Q-cohomology on operators contains interfaces wrapped on E_τ.

  [We will observe a relation between Q and Q_A]

BPS equations for Q

$$\underset{z}{E_\tau} \times \underset{y}{\mathbb{R}}$$

- Path integral localizes on supersymmetric configurations determined by the "BPS" eqns.

(1) $\partial W = 0 \Rightarrow \mu_{\mathbb{C}} = 0$ (complex moment map constraint)

(2) $(D_y + \sigma_{\mathbb{R}} + m_{\mathbb{R}})\phi = 0$ for every chiral (matter field)

(3) $D_y \sigma_{\mathbb{R}} = \mu_{\mathbb{R}} + [\sigma_{\mathbb{C}}, \bar{\sigma}_{\mathbb{C}}]$

(2), (3) } GRADIENT FLOW on $T^*R \times \mathfrak{g} \otimes \mathbb{R}^3 / G$

FOR $h = \underbrace{\mathrm{Tr}(\sigma_{\mathbb{R}} \mu_{\mathbb{R}})}_{\text{gauge action}} + \underbrace{\mathrm{Tr}(m_{\mathbb{R}} \mu^F_{\mathbb{R}})}_{\text{flavor action}} + \underbrace{\mathrm{Tr}\,\sigma_{\mathbb{R}}[\sigma_{\mathbb{C}}, \bar{\sigma}_{\mathbb{C}}]}_{\text{part of gauge action}}$

Additionally, there are equations that follow from equivariance (i.e. $Q^2\phi = 0$), such as: $D_{\bar{z}}\varphi = 0$ $\quad$ [they are less important]

Notice: we neither have stability condition, nor $\mu_{\mathbb{R}} = 0$.

! Instead, we have a gradient flow on $\mu_{\mathbb{C}}^{-1}(0)/G$.

For Morse function $h =$

If masses vanish, $m_{\mathbb{R}} = 0$, then fixed points of this flow have $\mu_{\mathbb{R}} = 0$, $\sigma_{\mathbb{R}} = 0$

$\Rightarrow$ we find $X_H = \{\mu_{\mathbb{C}}^{-1}(0) \cap \mu_{\mathbb{R}}^{-1}(0)\}/G$ there.

[If we ignore $\sigma_{\mathbb{R}}$, i.e. look at $T^*R \times \mathfrak{g}\otimes\mathbb{C}$, then gradient trajectories $\subset G_{\mathbb{C}}$-orbits]

If masses corresponding to $A \subset \mathbb{T}_F$ are turned on, then fixed points of this flow do not form the whole X_H.

We get $X_H^A = A$-fixed locus in X_H.

[Ignoring $\sigma_{\mathbb{R}}$, now gradient trajectories lie within $G_{\mathbb{C}} \times A$-orbits]

So the Higgs branch image of our gradient flow [impose stability & quotient by $G_{\mathbb{C}}$ $\xrightarrow{\text{induces}}$ flow on X_H]

$=$ orbits of the $\mathbb{C}^\times \subset A$ action on X_H, where $\mathbb{C}^\times$ is determined by masses.

The fixed points of this flow $= X_H$, i.e. torus fixed points.

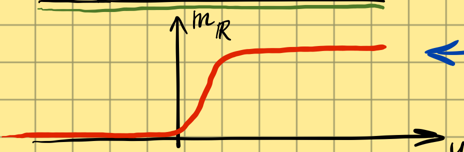
$\rightarrow$ All critical points of h have the same index $= \frac{1}{2}$ dim.

$\Rightarrow$ half-dimensional attractors $\longrightarrow$ leads to stable envelopes.

We will find that Stab is a map from the vector space of SUSY vacua at $m_{\mathbb{R}} \neq 0$ to the vector space of SUSY vacua at $m_{\mathbb{R}} = 0$. Realized via a SUSY interface that varies mass.

IV. Janus interface.

Can we interpolate between different values of $m_{\mathbb{R}}$ preserving Q?



$\longleftarrow$ Want something like this...

Standard physical approach: find a supersymmetric background!

Consider a background vector multiplet "gauging" flavor symmetry, $\mathcal{V}^F$

[add flavor connection $A_\mu^f dx^\mu$ and its superpartners, but don't integrate over them in the path integral.]

$$\begin{cases} \text{fermions} = 0 \\ Q(\text{fermions}) = 0 \end{cases} \qquad \mathcal{V}^F = (A_\mu^f,\ \sigma_{\mathbb{R}}^f,\ D_{\mathbb{R}}^f,\ \bar{\lambda}^f,\ \lambda^f)$$

Constant real mass corresponds to: $\sigma_{\mathbb{R}}^f = m_{\mathbb{R}}$; else $= 0$.

A simple generalization: $\boxed{\sigma_{\mathbb{R}} = m_{\mathbb{R}}(y);\ D_{\mathbb{R}} = i\, m_{\mathbb{R}}'(y)}$

$\rightarrow$ preserves 2d $\mathcal{N} = (0,2)$ SUSY (or even $(2,2)$, if we switch off $\hbar$).

the mass term is given by the Lagrangian: $\mathcal{L}_m = \bar{\varphi}\, m_{\mathbb{R}}(y)^2 \varphi + 2\bar{\varphi}\,\sigma_{\mathbb{R}} m_{\mathbb{R}}(y)\varphi - \bar{\varphi}\, m_{\mathbb{R}}'(y)\varphi + i\bar{\psi}\, m_{\mathbb{R}}(y)\psi$

what happens under: $m_{\mathbb{R}}(y) \rightarrow m_{\mathbb{R}}(y) + \delta m_{\mathbb{R}}(y)$?

Answer: in a general $\mathcal{N}=2$ theory, $\delta\mathcal{L}_m \neq 0$, details of $m_{\mathbb{R}}(y)$ are important...

But, in an $\mathcal{N}=4$ theory, $\delta\mathcal{L}_m = \{Q, \dots\}$ [+ something vanishing under correlators]

as long as $\delta m_{\mathbb{R}}(\pm\infty) = 0 \Rightarrow$ precise shape of $m_{\mathbb{R}}(y)$ doesn't matter!

- Pick $m_{\mathbb{R}}(-\infty)$, $m_{\mathbb{R}}(+\infty)$ and interpolate them in some way...

$\Rightarrow$ unique answer in Q-cohomology.

In fact, only chambers of $m_{\mathbb{R}}(-\infty)$ and $m_{\mathbb{R}}(+\infty)$ matter...

in the space $\mathfrak{a} \ni m_{\mathbb{R}}$ $\qquad \mathfrak{a} \setminus \bigcup_i \alpha_i^{\perp} = \bigsqcup_i C_i$

Physics provides us a unique (up to Q-exact terms) interface interpolating <u>between</u> $m_{\mathbb{R}}(-\infty) \in C_i$ and $m_{\mathbb{R}}(+\infty) \in C_j$

Recall that Q-cohomology of states in $\mathcal{H}[E_\tau]$ = space of vacua

Therefore we find in the Q-cohomology:

$$\mathcal{J}(m_{\mathbb{R}}(+\infty), m_{\mathbb{R}}(-\infty)) : \left\{\begin{matrix}\text{space of}\\ \text{vacua at}\\ m_{\mathbb{R}}(-\infty)\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}\text{space of}\\ \text{vacua at}\\ m_{\mathbb{R}}(+\infty)\end{matrix}\right\}$$

<u>Janus for the F.I. parameter:</u>

There is an analogous construction of a SUSY background with $\zeta_{\mathbb{R}}(y)$.

[FI term = mass for "topological symmetry"

If A = $U(1)$ gauge field, $F = dA \Rightarrow J^{top} = *F$ is conserved $d^* J^{top} = 0$]

$$\Rightarrow \text{Linear map } \tilde{\mathcal{J}}(\zeta_{\mathbb{R}}(+\infty), \zeta_{\mathbb{R}}(-\infty)) : \left\{\begin{matrix}\text{vacua}\\ \text{at } \zeta_{\mathbb{R}}(-\infty)\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}\text{vacua}\\ \text{at } \zeta_{\mathbb{R}}(+\infty)\end{matrix}\right\}$$

<u>Recall:</u> Higgs phase $\longleftrightarrow$ $\zeta_{\mathbb{R}} \gg 0$, $m_{\mathbb{R}} = 0$ $\qquad$ Coulomb phase $\longleftrightarrow$ $\zeta_{\mathbb{R}} = 0$, $m_{\mathbb{R}} \gg 0$

$$\Rightarrow \underbrace{\mathcal{J}(0, m_{\mathbb{R}})}_{} \underbrace{\tilde{\mathcal{J}}(\zeta_{\mathbb{R}}, 0)}_{} = \text{phase interface between Higgs \& Coulomb ph.}$$

Stable envelope for X_H

can be understood in the original theory via BPS eqn's (gradient flow)

Interesting in the Higgs phase.

Stable envelope for X_C

Interesting in the Coulomb phase can be understood via BPS eqns in the mirror theory

Let us apply this.

Study of Higgs and Coulomb phases and transition between them $\longleftrightarrow$ Symplectic duality mathematically [Bullimore-Dimofte-Gaiotto-Hilburn]

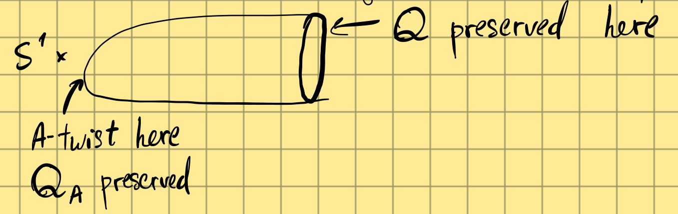
Vertex function = K-theoretic count of quasimaps (into X_H) [Okounkov '15] $\qquad V_\alpha \qquad K_T(X_H)$

Physically: partition function on $\underbrace{S^1 \times (\text{A-twist})}_{\langle C |} \leftarrow$ vacuum $|\alpha\rangle$ $\longleftarrow$ In the Higgs phase

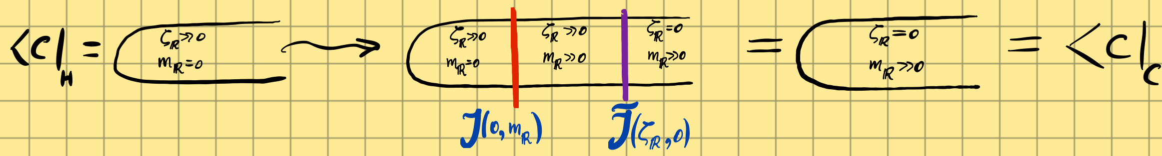
$V_\alpha = \langle C | \alpha \rangle$

[BPS eqn's: $\left.\begin{matrix} D_{\bar z}\phi = 0 \\ \mu_{\mathbb{R}} = e^2 F_{z\bar z}\end{matrix}\right\} \Rightarrow$ vortices $\longrightarrow$ quasimaps in the $e \to \infty$ limit]

There exists another background that computes the same:



Let us use it to understand what happens to V_α under the phase transition:



$$\langle C|_H \, \mathcal{J}(0, m_R)\, \tilde{\mathcal{J}}(\zeta_R, 0) = \langle C|_C$$

Rk: $m_R \gg 0$ or $\zeta_R \gg 0$ always means tending to infinity within a chosen chamber.

$\rightarrow$ Janus interfaces realize a phase transition, "transports" $\langle C|$ into Coulomb phase

$\langle C|_C$ gives "vertex function" that counts "quasimaps into the Coulomb branch"

Coulomb branch X_C is the Higgs branch $\tilde{X}_H$ of the 3d mirror theory.

If the mirror admits a quiver description, $\tilde{X}_H = X_C$ is a GIT quotient $\rightsquigarrow$ notion of quasimaps is clear

$\longrightarrow$ arrive at the fact known from [Aganagic-Okounkov],

that Elliptic Stable envelope = transition matrix that relates V_α of X_H to $\tilde{V}_\alpha$ of $\tilde{X}_H$. (3d mirror)

More generally, X_C might not admit a GIT description, yet our picture with phase transition still holds.

$\Rightarrow$ Elliptic stable envelopes realize a phase transition that relates vertex functions of the Higgs & Coulomb branches.

Problem: how to define quasimaps into the Coulomb branch and how to count them?

V. Vacua and cohomology.

Recall that $\{Q, Q^\dagger\} = H$, so $\{Q\text{-cohomology on } \mathcal{H}[E_\tau]\} \simeq \mathcal{V}$ – space of SUSY ground states.

- In 1d (i.e. in quantum mechanics), it is known that $\mathcal{V} = H^\bullet_T(X_H)$ [Witten]

  $Q = d + \iota_v$, $Q^\dagger = d^* + v^\flat \wedge$ $\longrightarrow$ Q-cohomology = equivariant de Rham of X_H.

- Lift to 2d: H_T is lifted to K_T

It is natural to think of a 2d theory $(\underbrace{\text{NLSM into } X_H}_{\text{low-energy description}})$ on $S^1 \times \mathbb{R}$ as a 1d NLSM into LX_H.

$$Q^2 = \underbrace{2iD_\varphi}_{S^1 \text{ rotation}} + \underbrace{m}_{\text{flavor transformation}} \Longrightarrow H_{T\times S^1}(LX_H)$$

$\nwarrow$ loop rotation.

- Lift to 3d: $Ell_T(X_H)$ Similar story:

  $Q^2 = 2iD_{\bar{z}}$; 3d NLSM into X_H is like 2d NLSM into LX_H, with extra equivariance w.r.t. loop rotations. $\Longrightarrow$ $\underset{\substack{\text{this is } Q\text{-cohomology}\\ \downarrow}}{K_{T\times S^1}(LX_H)} \simeq$ "$Ell_T(X_H)$"

Let us take the perspective as in e.g. [A-O] $Ell_T(X_H)$ is a scheme over

$$\underbrace{\mathcal{E}_T = T / q^{\mathrm{cochar}(T)}}_{\text{flat } T\text{-connections on } E_\tau \,=\, \text{elliptic equivariant parameters}}$$

If for generic equivariant parameters, there are only isolated massive vacua, then

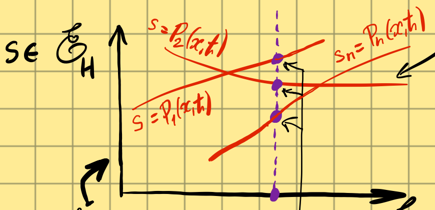
$\to$ $Ell_T(X_H)$ can be understood as the classical description of such vacua.

We have equivariant parameters $(x,\hbar)\in\mathcal{E}_T$ for $T = A\times\mathbb{C}^\times_\hbar$ and "gauge parameters", or Chern roots $s\in\mathcal{E}_H$,

Here $\mathcal{E}_H = H_{\mathbb{C}}/q^{\mathrm{cochar}(H)}$, H = maximal torus of the gauge gp. G

Physically, $(x,\hbar)$ and s are T- and H-flat connections on E_τ.

In each massive vacuum, s adjust itself to a value $s_i = P_i(x,\hbar)$, which screens some x and $\hbar$, and allows a matter vev:



this graph is $Ell_T(X)$

"Classical spectral manifold"

Massive vacua for a fixed value of equivariant parameters $(x,\hbar)\in\mathcal{E}_T$

Irreducible components $\leftrightarrow$ massive vacua

E.g. for $SQED_n$, $X_H = T^*\mathbb{C}P^{n-1}$, Ell is given by:

$$\prod_{i=1}^n (s x_i \hbar^{1/2} - 1) = 0$$

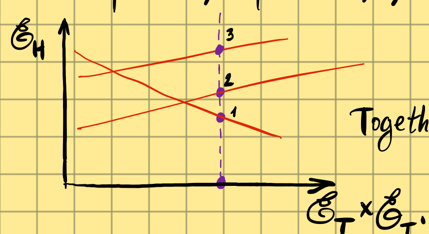
U(1) gauge theory with n hypermultiplets

This is really Spec $K_T(X_H)$. Ell is obtained mod q

In the quantum case, vacua are states, i.e. elements of the vector space.

So in the picture, points 1, 2, 3 correspond to 1-dimentional vector spaces

$\mathbb{C}|1\rangle$, $\mathbb{C}|2\rangle$, $\mathbb{C}|3\rangle, \dots$ (quantum vacua)



Together, they add up to a fiber of a vector bundle over $\mathcal{E}_T\times\mathcal{E}_{T'}$ – the bundle of vacua.

$T' = \mathrm{Pic}_T(X_H)$

Here we replaced $\mathcal{E}_T$ by $\mathcal{E}_T\times\mathcal{E}_{T'}$, and $Ell_T(X_H)$ by $E_T(X_H) = Ell_T(X_H)\times\mathcal{E}_{T'}$

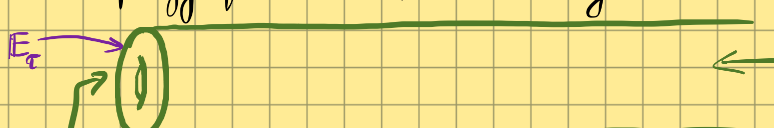
$\mathcal{E}_{T'}$ – "Kähler parameters" z. They are flat connections for T^{top} on E_τ and must be included in quantum case.

In fact, it is better to think of a line bundle:

$$\begin{array}{c}\mathcal{L}\\ \downarrow \\ E_T(X) = Ell_T(X_H)\times\mathcal{E}_{T'}\end{array}$$

Then the bundle of vacua is a pushforward of $\mathcal{L}$ with respect to $\begin{array}{c}E_T(X_H)\\ \downarrow\\ \mathcal{E}_T\times\mathcal{E}_{T'}\end{array}$.

The topology of $\mathcal{L}$ is determined by the 't Hooft anomalies in QFT.



$\leftarrow$ isolated vacuum α

read off "wave functional" corresponding to α here.

[L on T^*R]

$\to$ To construct $\mathcal{H}[E_\tau]$, need to choose polarization on the phase space.

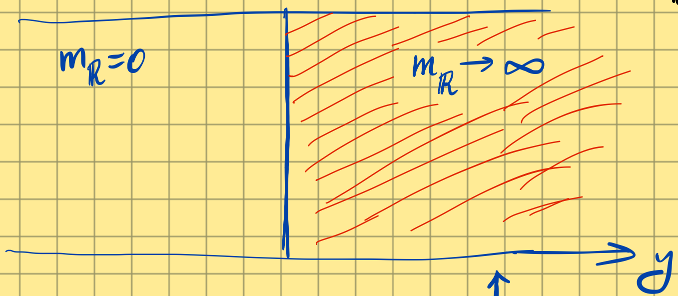
$\to$ Want SUSY, (2,2) polarization: comes from polarization on X_H (as holo. symp. mnfld.)

$\mathcal{L}$ has sections $\sim \prod_{w,f \in L} \frac{\vartheta(s^w x^f \hbar^{1/2})}{\vartheta(s^w x^f)\vartheta(\hbar^{1/2})} \times \left(\frac{\vartheta(\hbar)}{\vartheta(\hbar^{1/2})^2}\right)^{|G|} \times \frac{\vartheta(sz)}{\vartheta(s)\vartheta(z)}$

[work in progress]

VI. Effective 2d description of stable envelopes.

Consider mass Janus $J(0, m_{\mathbb{R}})$ for $m_{\mathbb{R}} \longrightarrow \infty$ in C.



- This realizes

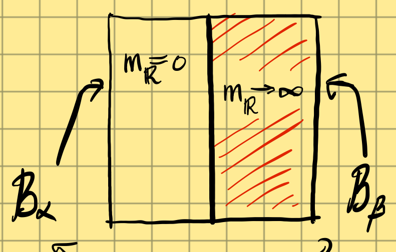
$$\mathrm{Stab}: \begin{Bmatrix}\text{vacua}\\ \text{at } m_{\mathbb{R}} \gg 0\end{Bmatrix} \longrightarrow \begin{Bmatrix}\text{vacua}\\ \text{at } m_{\mathbb{R}}=0\end{Bmatrix}$$

$$\Gamma(E_T(X_H^A), \mathcal{L}) \longrightarrow \Gamma(E_T(X_H), \mathcal{L})$$

- BPS eqn's = gradient flow correspond to the construction in math.

- Since mass is $m_{\mathbb{R}} \to \infty$ here, the flow happens really fast. I.e., we don't need infinite time.

Enough to consider an interval:



$\Longrightarrow$ Partition function on $E_\tau \times I$.

$\downarrow$

"Interval reduction"

$\downarrow$

This leads to an effective 2d (0,2) description on E_τ.

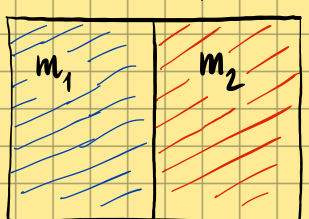
Elliptic genus = matrix element of Stab.

This part is in progress, but one can write quite general formulas for Stab in this way.

VII. R-matrices

Consider $J(m_1, m_2) = J(m_1, 0) \circ J(0, m_2) = \mathrm{Stab}^{-1}_{C_1} \circ \mathrm{Stab}_{C_2}$

were $m_1 \in C_1$, $m_2 \in C_2$, m_1 and m_2 are very large.



In 1d QFT case, we get a map

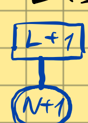
$$H_T(X^A) \longrightarrow H_T(X^A).$$

- The interface supports some trapped modes.

- Such interphaces can "change" the gauge group via Higgs mechanism.

E.g. Consider $U(N+1)$ gauge theory w/ $L+1$ fundam. hypers.

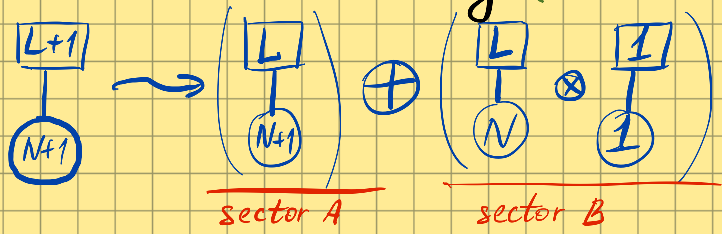
$X_H = T^* Gr(N+1, L+1)$.



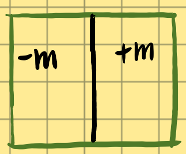
Consider mass for $U(1)$ that rotates the last hyper.

$$X_H^{U(1)} = T^*Gr(N+1, L) \sqcup T^*Gr(N, L)$$

At the QFT level, the mass breaks theory (at low energies) into subsectors:



Can consider an interface



$\Rightarrow$ Leads to a 2×2 block R-matrix $\begin{array}{c c} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} * & * \\ * & * \end{bmatrix} \end{array}$ that talks between these two sectors.
[Maulik-Okounkov]

To be continued... (see more in upcoming paper(s))