Knot Categorification from Mirror Symmetry

Part III

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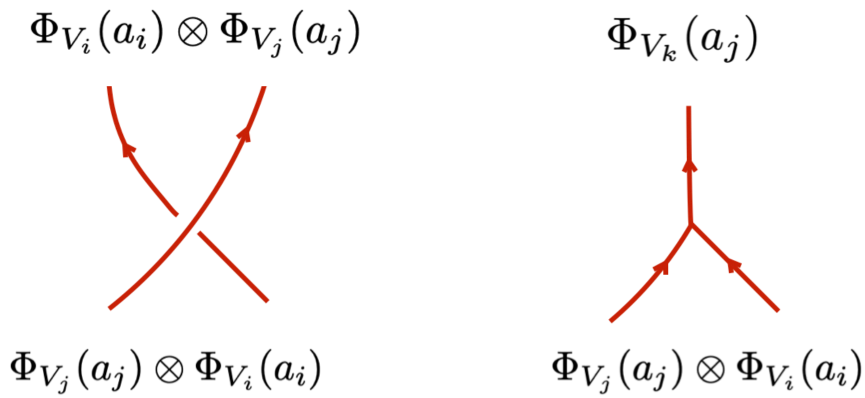
Chapter V

Homological link invariants from

$\mathcal{X}$

We understood the geometric origin of



the basic structures in conformal field theory:

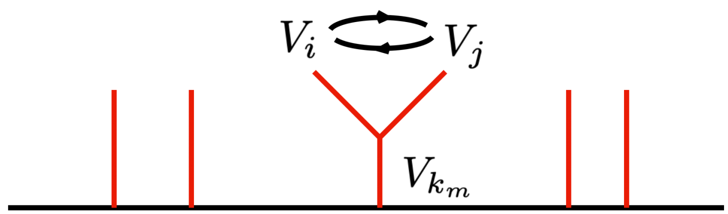
fusion and braiding

from

$$\mathscr{D}_{\mathcal{X}} = D^b Coh_{\mathrm{T}}(\mathcal{X})$$

We saw that the analogue of the fact that

"fusion diagonalizes braiding"



is a perverse filtration, in the sense of Rouquier and Chuang,

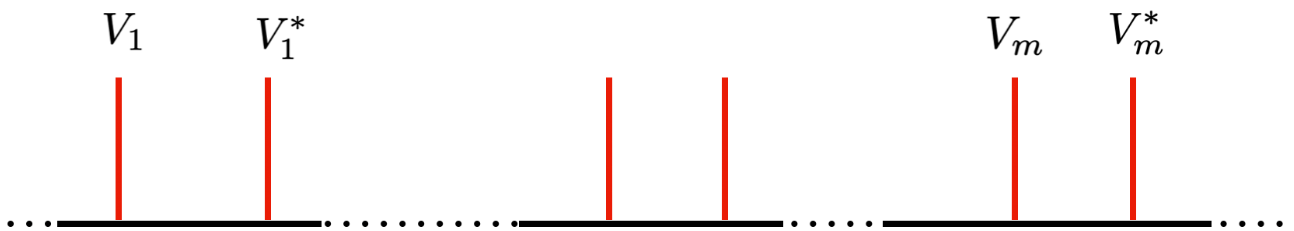
$$\mathscr{D}_{k_0} \subset \mathscr{D}_{k_1} \ldots \subset \mathscr{D}_{k_{max}} = \mathscr{D}_{\mathcal{X}}$$

whose terms are labeled by fusion products

$$V_i \otimes V_j = \bigoplus_{m=0}^{max} V_{k_m}$$

which is preserved by the action of braiding on $\mathscr{D}_{\mathcal{X}}$

We saw that, when a collection of vertex operators come together in pairs of minuscule representations and their conjugates



our manifold has a local neighborhood where we can approximate it as

$$\mathcal{X} \sim T^*U$$

where

$$U = U_1 \times \ldots \times U_m = G/P_1 \times \ldots \times G/P_m$$

is a product of minuscule Grassmannians.

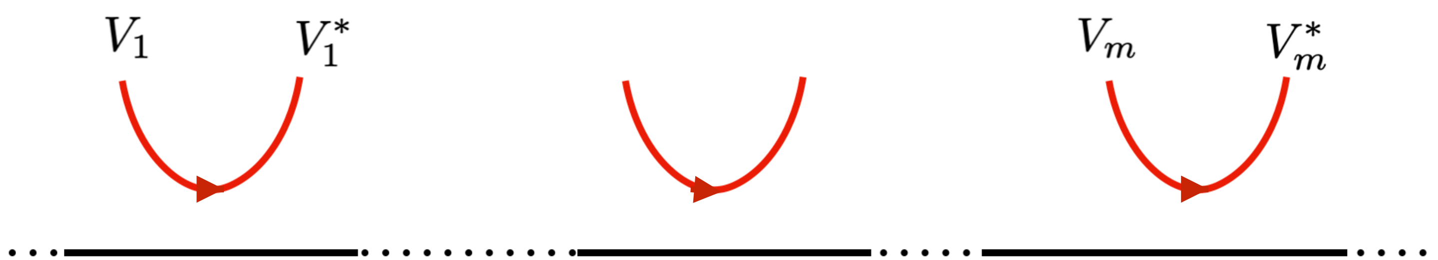
We found a very special B-type brane

$$\mathcal{U} \in \mathscr{D}_{\mathcal{X}}$$

which is the structure sheaf of this vanishing cycle,

$$\mathcal{U} = \mathcal{O}_U$$

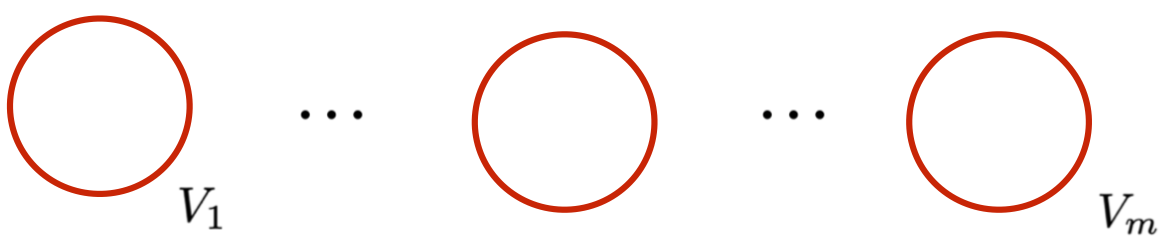
The vertex function of this brane is the conformal block



and

$$\mathrm{Hom}^{*,*}_{\mathscr{D}_{\mathcal{X}}}(\mathcal{U}, \mathcal{U})$$

categorifies the quantum group invariant of a product of unknots:



The homology group

$$Hom^{*,*}(\mathcal{U}, \mathscr{B}\mathcal{U})$$

is a braid invariant whose Euler characteristic

$$\chi(\mathcal{U}, \mathscr{B}\mathcal{U}) = \sum_{n,k\in\mathbb{Z}} (-1)^n \mathfrak{q}^{k-D/2} \dim Hom(\mathcal{U}, \mathscr{B}\mathcal{U}[n]\{k\})$$

is the matrix element

$$\chi(\mathcal{U}, \mathscr{B}\mathcal{U}) = (\mathfrak{U}, \mathfrak{B}\mathfrak{U})$$

of the corresponding braiding matrix $\mathfrak{B} \in U_{\mathfrak{q}}({}^L\mathfrak{g})$

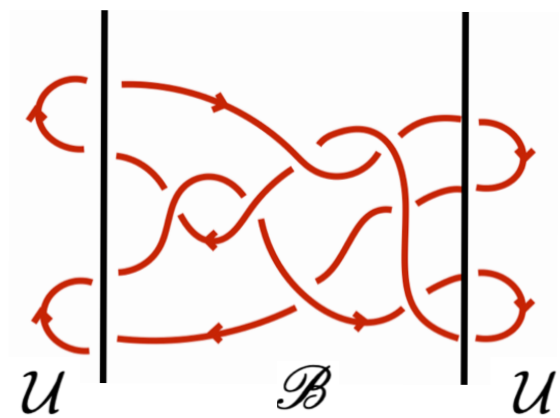
Using very special properties of perverse filtrations

and these vanishing cycle branes

it is not hard to show that not only do the homology groups

$$Hom^{*,*}_{\mathscr{D}_{\mathcal{X}}}(\mathscr{B}\mathcal{U},\mathcal{U})$$

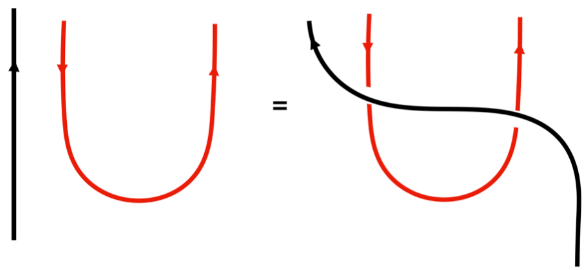
not only manifestly categorify the corresponding $U_{\mathfrak{q}}({}^L\mathfrak{g})$ link invariants,



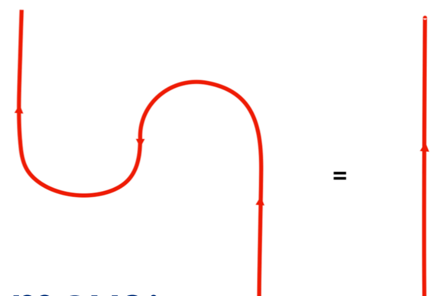
they are themselves link invariants.

To show this, additional relations must hold:

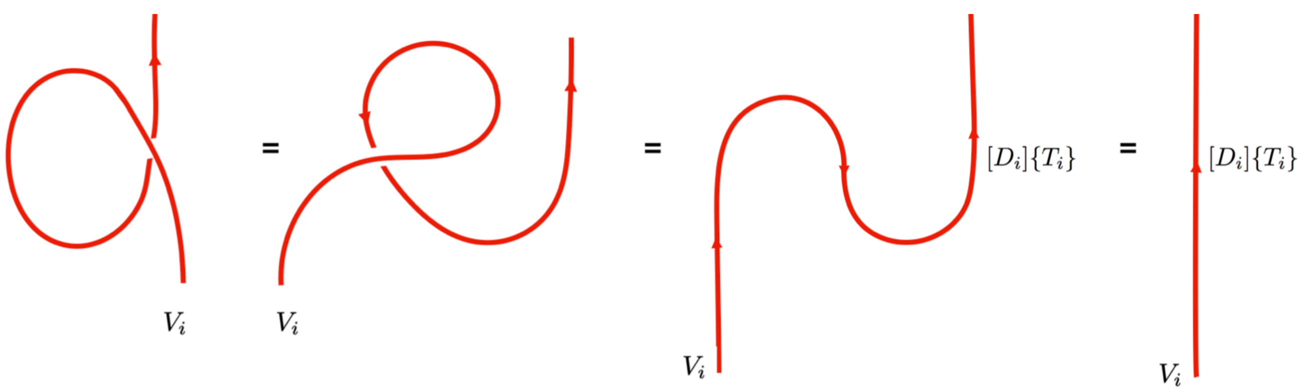
1. A version of ``pitchfork” identity:



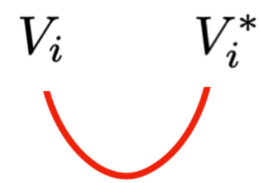
2. Reidermeister 0 or “S-move”:



3. Framed Reidermeister I move:



For all of these, in conformal field theory, one wants to view a cap



as a map between the space of conformal blocks of the form

$$\langle\lambda|\Phi_{V_1}(a_1)\cdots\ \mathbb{1}\ \cdots\Phi_{V_n}(a_n)|\lambda'\rangle$$

which come from $\mathcal{X}_{n-2} = \mathrm{Gr}_{\nu}^{\vec{\mu}_{n-2}}$

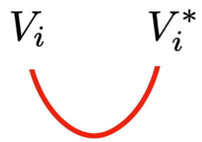
and the space of conformal blocks obtained by pair creation

$$\mathfrak{C}_i : \quad \mathbb{1} \quad \rightarrow \quad \Phi_{V_i}(a_{2i-1}) \otimes \Phi_{V_i^*}(a_{2i})$$

which come from $\mathcal{X}_n = \mathrm{Gr}_{\nu}^{\vec{\mu}_n}$ and look like

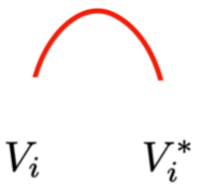
$$\langle\lambda|\Phi_{V_1}(a_1)\cdots\ \Phi_{V_i}(a_{2i-1}) \otimes \Phi_{V_i^*}(a_{2i})\ \cdots\Phi_{V_n}(a_n)|\lambda'\rangle$$

This pair creation



$$\mathfrak{C}_i : \quad \mathbb{1} \quad \rightarrow \quad \Phi_{V_i}(a_{2i-1}) \otimes \Phi_{V_i^*}(a_{2i})$$

has the inverse process



$$\mathfrak{C}_i^{\vee} : \quad \Phi_{V_i}(a_i) \otimes \Phi_{V_i^*}(a_j) \quad \rightarrow \quad \mathbb{1}$$

where a pair of vertex operators come together and disappear.

The fact that these maps in conformal field theory satisfy

the three relations we just named

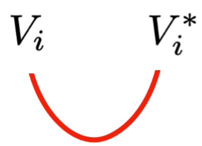
follows from properties of fusion and braiding in conformal field theory.

The fact they hold is the algebraic reason

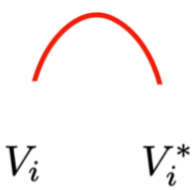
why conformal field theory gives rise to link invariants.

These maps between the spaces of conformal blocks

originate from functors



$$\mathscr{C}_i : \mathscr{D}_{\mathcal{X}_{n-2}} \longrightarrow \mathscr{D}_{\mathcal{X}_n} \quad \longmapsto \quad \mathfrak{C}_i : \ \mathbb{1} \ \to \ \Phi_{V_i}(a_{2i-1}) \otimes \Phi_{V_i^*}(a_{2i})$$



$$\mathscr{C}_i^\vee : \mathscr{D}_{\mathcal{X}_n} \longrightarrow \mathscr{D}_{\mathcal{X}_{n-2}} \quad \longmapsto \quad \mathfrak{C}_i^\vee : \ \Phi_{V_i}(a_i) \otimes \Phi_{V_i^*}(a_j) \ \to \ \mathbb{1}$$

between derived categories.

These functors can be defined in the standard way,

via a Fourier-Mukai kernel

$$\mathcal{C}_i \in \mathscr{D}(\mathcal{X}_{n-2} \times \mathcal{X}_n)$$

which is a structure sheaf

$$\mathcal{C}_i = \mathcal{O}_{C_i}$$

of a holomorphic Lagrangian C_i on the product $\mathcal{X}_{n-2} \times \mathcal{X}_n$

which can be used to map the objects and morphisms between them.

The existence of perverse filtrations

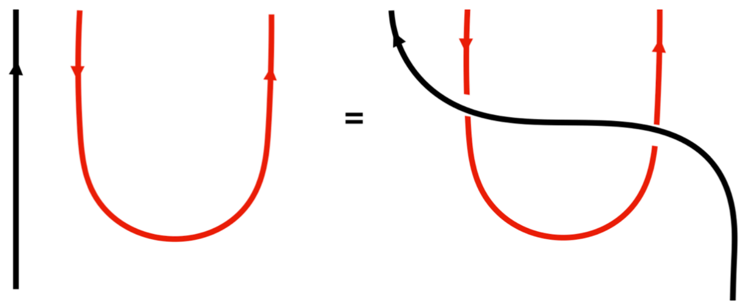
on the derived category

and their relation to conformal field theory

provides an apriori way to understand why

these relations hold in the derived category.

For example,



states that we get a derived equivalence

$$\mathscr{B} \circ \mathscr{C}_i \cong \mathscr{C}_i''$$

where

$$\mathscr{C}_i'' : \mathscr{D}_{\mathcal{X}_{n-2}} \longrightarrow \mathscr{D}_{\mathcal{X}_n''} \qquad \mathscr{C}_i : \mathscr{D}_{\mathcal{X}_{n-2}} \longrightarrow \mathscr{D}_{\mathcal{X}_n}$$

are the cap functors on the left and the right and

$$\mathscr{B} : \mathscr{D}_{\mathcal{X}_n} \cong \mathscr{D}_{\mathcal{X}_n''}$$

corresponds to braiding $\Phi_{V_k}(a_k)$ with $(\Phi_{V_i}(a_i) \otimes \Phi_{V_i^*}(a_j))$

We can identify

$\mathscr{C}_i \mathscr{D}_{\mathcal{X}_{n-2}} \subset \mathscr{D}_{\mathcal{X}_n}$ and $\mathscr{C}_i'' \mathscr{D}_{\mathcal{X}_{n-2}} \subset \mathscr{D}_{\mathcal{X}_n''}$

as the bottom most parts of double filtrations of

$\mathscr{D}_{\mathcal{X}_n}$ and $\mathscr{D}_{\mathcal{X}_n''}$

which one gets near the intersection of a pair of walls

where three vertex operators come together.

The functor $\mathscr{B} : \mathscr{D}_{\mathcal{X}_n} \cong \mathscr{D}_{\mathcal{X}''_n}$ acts on the bottom parts of the double filtration only by degree shifts.

These turn out to be trivial in our case
or otherwise the relation we are trying to prove would not hold
even in conformal field theory.

The functor all of whose degree shifts are identity acts trivially
so one finds

$$\mathscr{B} \circ \mathscr{C}_i \cong \mathscr{C}''_i$$

An elementary consequence

is a new geometric explanation for

mirror symmetry of $U_{\mathfrak{q}}({}^L\mathfrak{g})$ link invariants

which states that the invariants of

a link K and its mirror image K^*

are related by

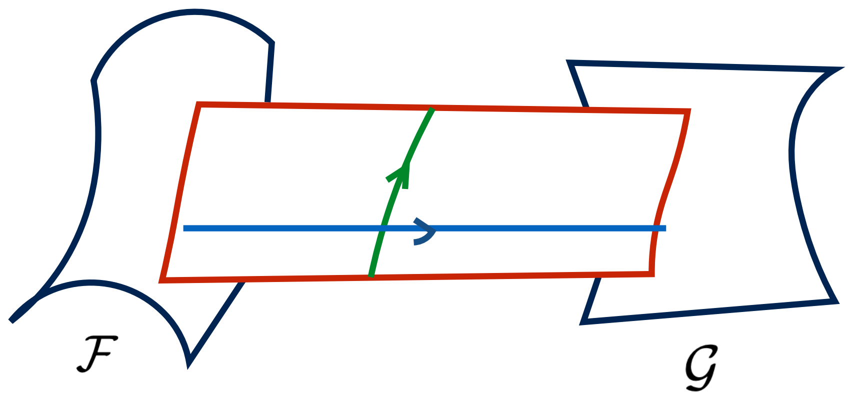
$$\mathcal{J}_K(\mathfrak{q}) = \mathcal{J}_{K^*}(\mathfrak{q}^{-1})$$

For us this follows from Serre duality

$$Hom_{\mathscr{D}_{\mathcal{X}}}(\mathcal{F}, \mathcal{G}[n]\{k\}) = Hom_{\mathscr{D}_{\mathcal{X}}}(\mathcal{G}, \mathcal{F}[2D-n]\{D-k\})$$

which is an isomorphism of Q-cohomology

with branes $\mathcal{F}$ and $\mathcal{G}$ at the two ends



and Q -cohomology obtained by a reflection that exchanges the endpoints.

The shift in the equivariant degree comes from the fact that, while $K_{\mathcal{X}}$

is trivial its unique holomorphic section is not invariant under $\mathbb{C}^{\times}_{\mathfrak{q}}$.

Taking the equivariant Euler characteristic

$$\chi(\mathcal{F},\mathcal{G}) = \sum_{n\in\mathbb{Z},k\in\mathbb{Z}^{\mathrm{rkT}}} (-1)^n \mathfrak{q}^{k-D/2} \dim\, Hom_{\mathscr{D}_{\mathcal{X}}}(\mathcal{F},\mathcal{G}[n]\{k\})$$

of both sides of

$$Hom_{\mathscr{D}_{\mathcal{X}}}(\mathcal{U},\mathscr{B}\mathcal{U}[n]\{k\}) = Hom_{\mathscr{D}_{\mathcal{X}}}(\mathscr{B}\mathcal{U},\mathcal{U}[2D-n]\{D-k\})$$

and using

$$\chi(\mathcal{U},\mathscr{B}\mathcal{U})(\mathfrak{q}) = J_K(\mathfrak{q}) \qquad \chi(\mathscr{B}\mathcal{U},\mathcal{U})(\mathfrak{q}) = J_{K^*}(\mathfrak{q})$$

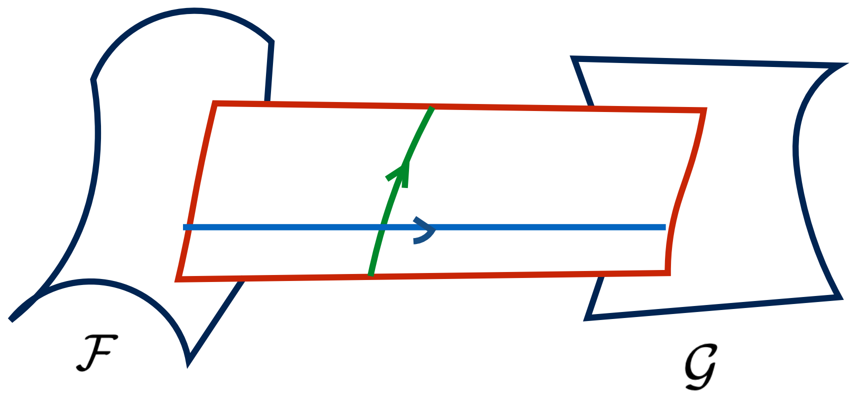
one finds

$$\mathcal{J}_K(\mathfrak{q}) = \mathcal{J}_{K^*}(\mathfrak{q}^{-1})$$

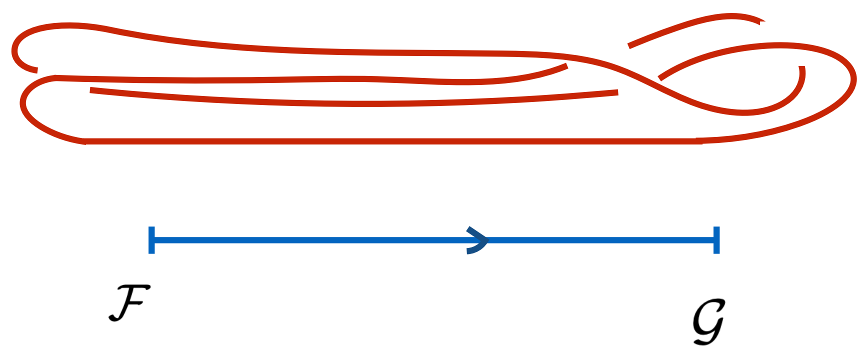
The fact that Serre duality implies mirror symmetry

$$\mathcal{J}_K(\mathfrak{q}) = \mathcal{J}_{K^*}(\mathfrak{q}^{-1})$$

is not an accident since the directions along the interval



and along the link, which get reflected, coincide.



Recently, Ben Webster proved that link invariants

$$Hom^{*,*}_{\mathscr{D}_{\mathcal{X}}}(\mathscr{B}\mathcal{U},\mathcal{U})$$

that come from

$$\mathscr{D}_{\mathcal{X}} = D^b Coh_{\mathrm{T}}(\mathcal{X})$$

in the way I described

are equivalent to invariants he defined in '13

KLRW algebras studied by

Khovanov and Lauda, by Rouquier and by himself.

As stated, neither the approach by

$$\mathscr{D}_{\mathcal{X}} = D^b Coh_{\mathrm{T}}(\mathcal{X})$$

nor by KRLW algebras is very computation friendly.

I will next describe how to reformulate the problem,

to get a much simpler description.

Chapter VI

The "equivariant mirror" of

$\mathcal{X}$

The second description is based on a Landau-Ginsburg model

which is "the equivariant mirror" of

$$\mathcal{X} = \mathrm{Gr}^{\vec{\mu}}{}_{\nu}$$

Ordinary, non-equivariant mirror of

$$\mathcal{X}$$

is a hyper-Kahler manifold

$$\mathcal{Y}$$

which is, to a first approximation,

given by a hyper-Kahler rotation of $\mathcal{X}$

As $\mathcal{X}$ has only Kahler but not complex moduli,

due to the T- equivariance we impose,

(since we took all the singular monopoles

to be at the origin of $\mathbb{C}$ in $\mathbb{R}^3 = \mathbb{R} \times \mathbb{C}$)

$\mathcal{Y}$ has only complex but no Kahler moduli turned on.

A description based on

$$\mathcal{Y}$$

would give a symplectic geometry approach to the categorification problem,

with

$$\mathscr{D}_{\mathcal{X}} = D^b Coh_{\mathrm{T}}(\mathcal{X})$$

replaced by its homological mirror,

an appropriate derived category of Lagrangian branes on $\mathcal{Y}$

We will take advantage of the fact that,

since we want to work equivariantly with respect to the

$$\mathbb{C}^\times_{\mathfrak{q}} \subset \mathrm{T}$$

action on $\mathcal{X}$ which scales its holomorphic symplectic form

$$\omega^{2,0} \to \mathfrak{q}\,\omega^{2,0}$$

all the relevant information about the geometry of $\mathcal{X}$

is contained in the locus preserved by this action.

The locus preserved by $\mathbb{C}^\times_{\mathfrak{q}}$ action on $\mathcal{X}$

$$X = \mathcal{X}|_{\mathbb{C}^\times_{\mathfrak{q}}}$$

is a holomorphic Lagrangian in $\mathcal{X}$ since it is mid-dimensional and

$$\omega^{2,0}|_X = 0$$

We will call X the core of $\mathcal{X}$.

Viewing $\mathcal{X}$ as the moduli space of monopoles on

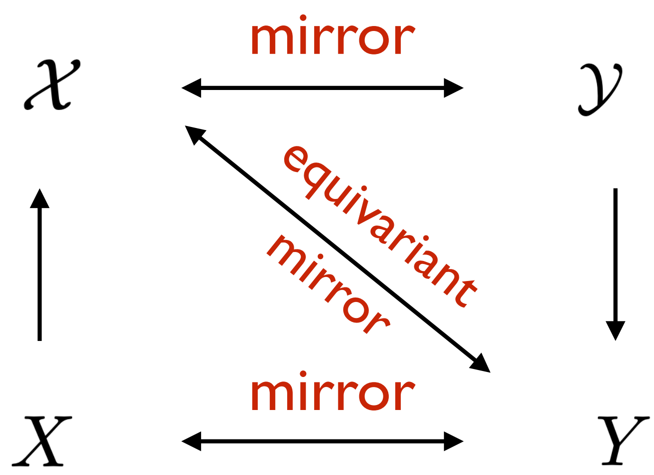
$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{C}$$

its core X is a locus in the moduli space where all the monopoles, singular or not, are at the origin of $\mathbb{C}$ and at points in $\mathbb{R}$

We will define the equivariant mirror of $\mathcal{X}$ which we will call

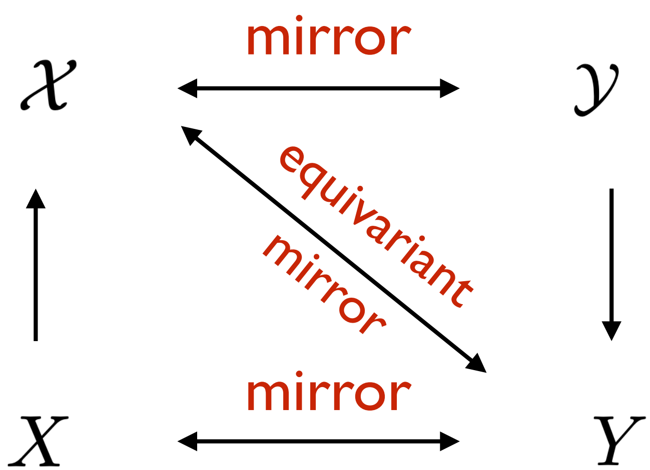
Y

to be the ordinary mirror of its core:



While X embeds into $\mathcal{X}$ as a

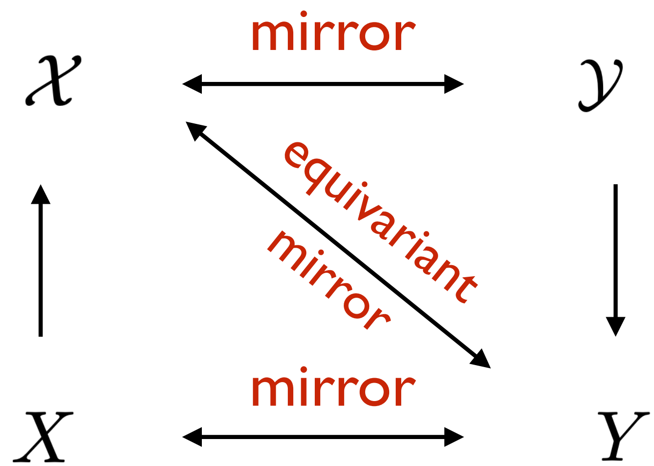
holomorphic Lagrangian submanifold of dimension $D = \dim_{\mathbb{C}} \mathcal{X}/2$



$\mathcal{Y}$ fibers over Y with holomorphic Lagrangian $(\mathbb{C}^{\times})^D$ fibers

The bottom row has as much information about the geometry as the top.

While the bottom row has as much information about the geometry as the top,



working downstairs as opposed to upstairs

will turn out to have many advantages which will slowly become manifest.

A model example to keep in mind is

$\mathcal{X}$ which is the resolution of an A_{m-1} surface singularity.

$\mathcal{X}$ is the moduli space of a single smooth $G = SU(2)/\mathbb{Z}_2$ monopole,

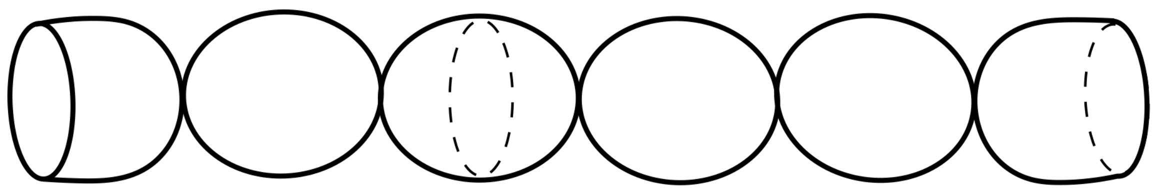
in presence of m singular ones.

For

$\mathcal{X}$ which is the resolution of an A_{m-1} surface singularity,

its core X looks like:

X



It is a collection of $m-1$ $\mathbb{P}^1$'s with a pair of infinite discs attached.

The ordinary mirror of $\mathcal{X}$ which is a resolution of

an A_{m-1} surface singularity,

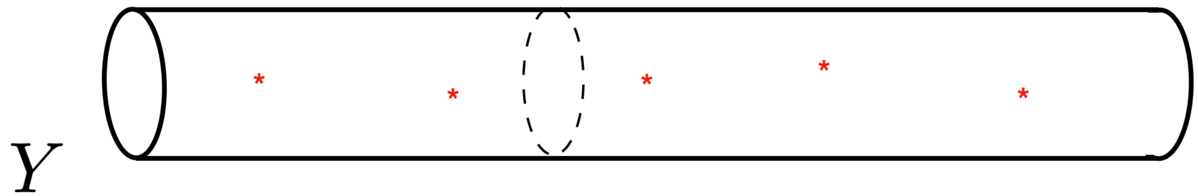
is $\mathcal{Y}$ which is a complex structure deformation of an “multiplicative”

A_{m-1} surface singularity,

with a potential which we will not need.

The multiplicative A_{m-1} surface $\mathcal{Y}$,

is a $\mathbb{C}^{\times}$-fibration over Y

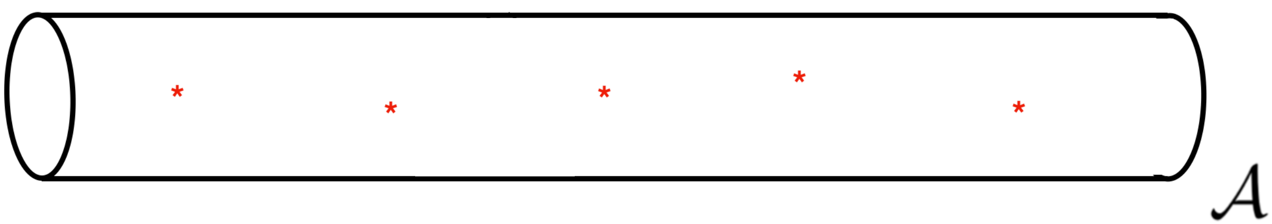


which is an infinite cylinder with m marked points in the interior.

At the marked points, the $\mathbb{C}^{\times}$ fibers of $\mathcal{Y}$ degenerate .

Y is a single copy of the Riemann surface

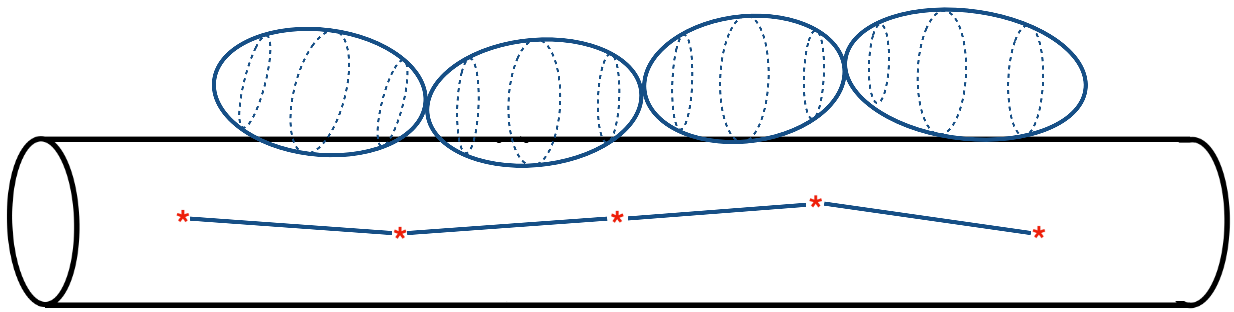
where the conformal blocks live:



The positions of punctures correspond to the marked points

where the $\mathbb{C}^{\times}$ fibration $\mathcal{Y} \to Y$ degenerates.

There are $m-1$ Lagrangian spheres in $\mathcal{Y}$

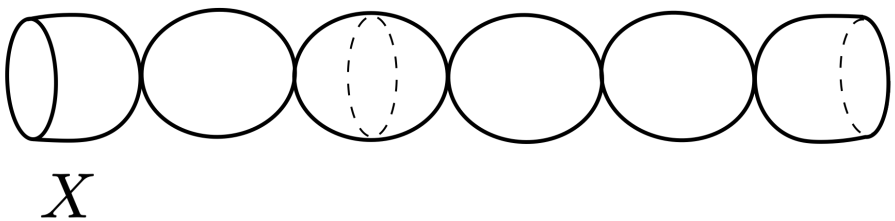


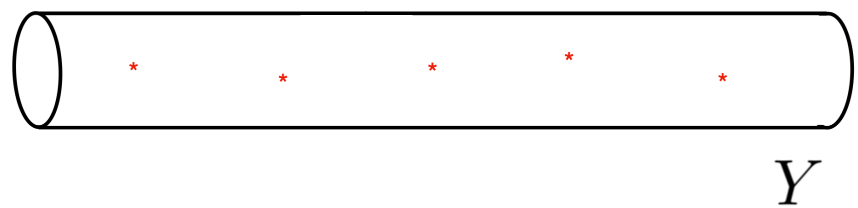
which are mirror to $m-1$ vanishing $\mathbb{P}^1$'s in $\mathcal{X}$.

They project to Lagrangians in Y that begin and end at the punctures.

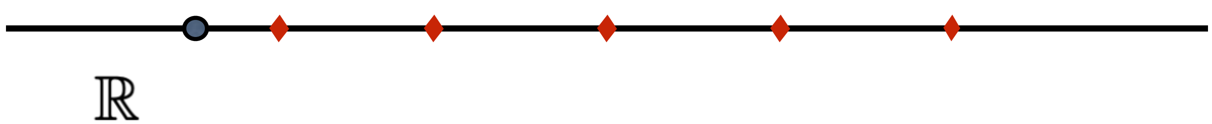
By SYZ mirror symmetry,

the mirror pair





share a common base,



which is the moduli space of one smooth $G = SU(2)/\mathbb{Z}_2$ monopole

on $\mathbb{R}$ (the locus in preserved by the $\mathbb{C}^{\times}_{\mathbf{q}}$ action on $\mathbb{R}^3 = \mathbb{R} \times \mathbb{C}$)

in presence of m singular ones.

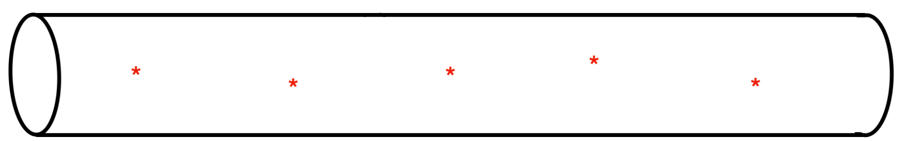
More generally, the equivariant mirror of

$$\mathcal{X} = Gr^{\vec{\mu}}{}_{\nu}$$

and the ordinary mirror of its core X, is

$$Y = \pi^*(Sym^{\vec{d}}(\mathcal{A})\backslash F_0)$$

where $\mathcal{A}$ is our Riemann surface with punctures,



$\vec{d} = (d_1, \ldots, d_{\mathrm{rk}})$ encodes the numbers of smooth monopoles, and where rk stands for the rank of $\mathfrak{g}$.

To the first approximation, the map π^* can be ignored.

The symmetrization in

$$Y = \pi^*(Sym^{\vec{d}}(\mathcal{A})\backslash F_0)$$

where

$$Sym^{\vec{d}}\mathcal{A} = \otimes_{a=1}^{\mathrm{rk}} Sym^{d_a}\mathcal{A}$$

comes from identifying the smooth monopoles

whose charge is associated to the same simple root

$$\text{weight } \nu = \underbrace{\text{highest weight}}_{\text{singular}} \ \mu - \underbrace{\sum_{a=1}^{\mathrm{rk}} d_a\, {}^L e_a}_{\text{smooth}} \geq 0$$

Projecting to the common SYZ base of

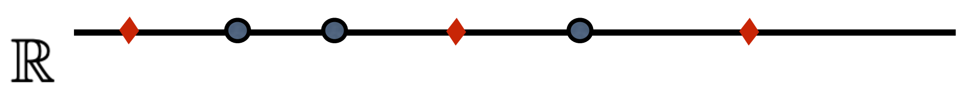
X and of Y

is the same as projecting $\mathcal{X}$,

the moduli space of singular monopoles on

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{C}$$

to



Including an equivariant $\mathrm{T}-$ action

on $\mathcal{X}$ and on X

corresponds to adding to the sigma model on

Y

a potential,

$$W = W^0/\kappa + \sum_{a=1}^{\mathrm{rk}} \lambda_a W^a$$

which is a multi-valued holomorphic function,

$$W^0 = \pi^*(\ln f^0) \qquad \text{and} \qquad W^a = \pi^*(\ln \prod_{\alpha} y_{a,\alpha})$$

The W^0 term in the potential

$$W = W^0/\kappa + \sum_{a=1}^{\mathrm{rk}} \lambda_a W^a$$

is mirror to the $\mathbb{C}^\times_{\mathbf{q}}$ equivariant action

$$W^0 = \pi^*(\ln f^0)$$

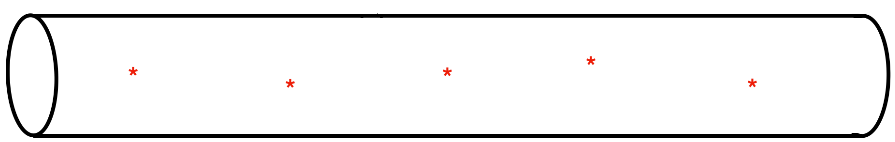
The function f^0 in

$$W^0 = \pi^*(\ln f^0)$$

is given by

$$f^0(y) = \prod_{a=1}^{\mathrm{rk}} \prod_{\alpha=1}^{d_a} \frac{\prod_i (1 - a_i/y_{\alpha,a})^{\langle {}^L e_a, \mu_i \rangle}}{\prod_{(b,\beta)\neq(a,\alpha)} (1 - y_{\beta,b}/y_{\alpha,a})^{\langle {}^L e_a, {}^L e_b \rangle/2}}$$

where each y is a coordinate on $\mathcal{A}$ viewed as a punctured complex plane.

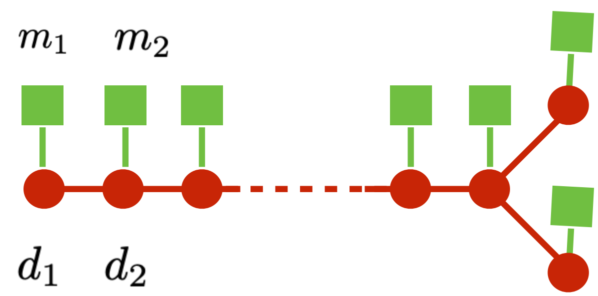
$\mathcal{A}$ 

Thinking of

$$\mathcal{X}$$

as the Coulomb branch of a three dimensional gauge theory

$\mathfrak{g} = D_n$



the term

$$W^0 = \pi^*(\ln f^0)$$

is associated to integrating out charged matter,

as I will explain in the last lecture.

The divisor F^0 in

$$Y = \pi^*(Y_0 \backslash F^0)$$

is the locus we need to remove from

$$Y_0 = Sym^{\vec{d}}\mathcal{A} = \otimes_{a=1}^{\mathrm{rk}} Sym^{d_a}\mathcal{A}$$

to define

$$W^0 = \pi^*(\ln f^0)$$

F^0 is the divisor of zeros and poles of the holomorphic function f^0

The W^a terms in the potential

$$W = W^0/\kappa + \sum_{a=1}^{\mathrm{rk}} \lambda_a W^a$$

are mirror to the Λ action in

$$\mathrm{T} = \Lambda \times \mathbb{C}_{\mathfrak{q}}^{\times}$$

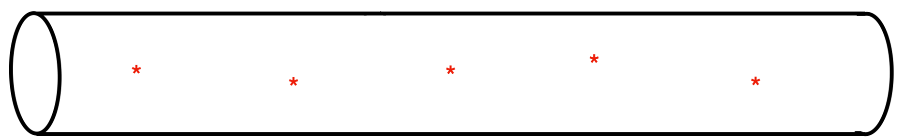
that preserves the holomorphic symplectic form.

In terms of Chern-Simons theory on

$$\mathbb{R}^2 \times S^1$$

$\vec{\lambda}$ is the conjugacy class of holonomy around the S^1

Since the $y = 0, \infty$ are already deleted from $\mathcal{A}$

$\mathcal{A}$ 

to define

$$W^a = \pi^*(\ln \prod_{\alpha} y_{a,\alpha})$$

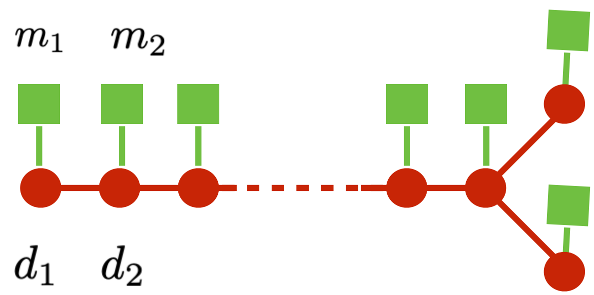
we do not need to delete any additional loci.

Thinking of

$$\mathcal{X}$$

as the Coulomb branch of a three dimensional gauge theory

$\mathfrak{g} = D_n$



the W^a terms in the potential

$$W = W^0/\kappa + \sum_{a=1}^{\mathrm{rk}} \lambda_a W^a$$

come from Fayet-Iliopolus terms.

Symplectic form on

$$Y$$

is inherited from symplectic form upstairs, on

$$\mathcal{Y} = (\mathbb{C}^\times)^D \to Y$$

by restricting to the vanishing $(S^1)^D$ in each $(\mathbb{C}^\times)^D$ fiber over Y

Both and are smooth, exact symplectic manifolds.

The Kahler form inherited from $\mathcal{Y}$ should provide

a sequence of blow ups that resolve the intersections

between the irreducible components of F^0 in Y_0 so

$$Y = \pi^*(Y_0 \backslash F^0)$$

which one can partially verify.

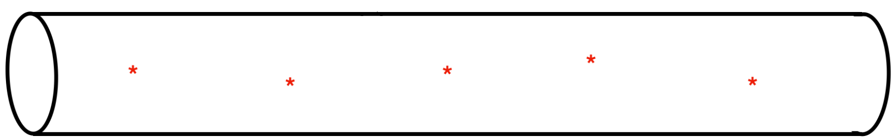
Finally,

$$2c_1(K_Y) = 0.$$

$K_Y^{\otimes 2}$ has a global holomorphic section $\Omega^{\otimes 2}$ where

$$\Omega = \bigwedge_{a=1}^{\mathrm{rk}} \Omega_a = \pi^* \Big(\bigwedge_{a=1}^{\mathrm{rk}} \bigwedge_{\alpha=1}^{d_a} \frac{dy_{\alpha,a}}{y_{\alpha,a}} \Big)$$

where each y is a coordinate on

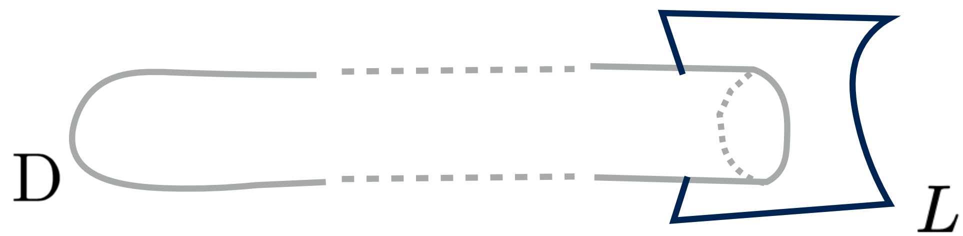
$\mathcal{A}$ 

viewed as a punctured complex plane.

From the mirror perspective, the conformal block of

$$\widehat{^L\mathfrak{g}}$$

is the partition function of the B-twisted theory on D ,



with A-type boundary condition at infinity, corresponding to a Lagrangian L in Y.

Such amplitudes have the following form

$$\mathcal{V}_\alpha[L] = \int_L \Phi_\alpha \, \Omega \, e^{-W}$$

where Ω is the top holomorphic form on Y ,

W is the Landau-Ginsburg potential,

and Φ 's are the chiral ring operators.

This reproduces the integral formulation of conformal blocks of

$$\widehat{{}^L\mathfrak{g}}$$

which goes back to work of Feigin and E.Frenkel in the ’80’s

and Schechtman and Varchenko.

We understand it here as a consequence of mirror symmetry.

The fact that the Knizhnik-Zamolodchikov equation which

the Landau-Ginzburg integral solves

$$\mathcal{V}_\alpha[L] = \int_L \Phi_\alpha \, \Omega \, e^{-W}$$

is also the quantum differential equation of $\mathcal{X}$ ....

…..gives a Givental type proof of 2d mirror symmetry

at genus zero, relating

the T-equivariant A-model on $\mathcal{X}$,

to

B-model on Y with potential W .

One of the harder problems in

a Landau-Ginsburg B-model with some target Y and potential

is identifying the "flat coordinates" on the moduli space,

in terms of which the amplitudes satisfy a simple set of equations.

$$\partial_i \mathcal{V}_\alpha - (C_i)_\alpha^\beta \mathcal{V}_\beta = 0.$$

where C_i is the matrix of multiplication by $\Phi_i = \partial_i W$,

and where $\partial_i = a_i \frac{\partial}{\partial a_i}$ is the ordinary derivative

with respect to the flat coordinates.

In the Landau-Ginsburg model,

the flat coordinates are obtained by solving a coupled set of equations

$$\partial_i W \cdot \Phi_\alpha = \sum_b (C_i)_\alpha^\beta \, \Phi_\beta + \sum_A \partial_A W \, \sigma_{i\alpha}^A$$
$$\partial_i \Phi_\alpha = \sum_A \partial_A \sigma_{i\alpha}^A$$

for the coordinates themselves, and operators Φ_α and $\sigma_{i\alpha}^A$.

In the present case,

the flat coordinates of the Landau-Ginsburg model are the relative positions

of vertex operators on $\mathcal{A}$,

which enter the Landau-Ginsburg potential.

The equation

$$\partial_i \mathcal{V}_\alpha - (C_i)_\alpha^\beta \mathcal{V}_\beta = 0.$$

the Knizhnik-Zamolodchikov equation.

There is a reconstruction theory,

due to Givental and Teleman, which says that

starting with the solution of quantum differential equation,

one gets to reconstruct all genus topological string amplitudes

of a semi-simple 2d field theory.

It follows that the B-twisted the Landau-Ginsburg model

$$(Y, W)$$

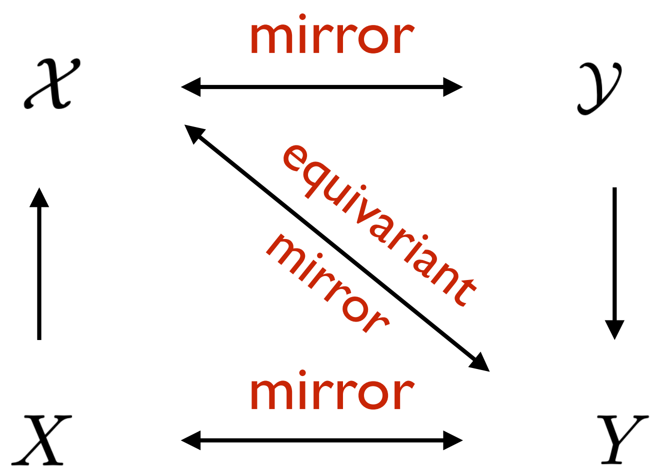
and A-twisted sigma model on

$$\mathcal{X}$$

working equivariantly with respect to T

are equivalent to all genus.

Thus, the equivariant mirror symmetry



holds as equivalence of topological string amplitudes.