Solutions

MATH 54 – LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS QUIZ 7B

1. (5 points) Let

$$x_1 = \begin{bmatrix} \cos(t) \\ 0 \\ 0 \end{bmatrix} \qquad x_2 = \begin{bmatrix} \sin(t) \\ \cos(t) \\ \cos(t) \end{bmatrix} \qquad x_3 = \begin{bmatrix} \cos(t) \\ \sin(t) \\ \cos(t) \end{bmatrix}$$

Compute the appropriate Wronskian. Is the set $\{x_1, x_2, x_3\}$ linearly independent or linearly dependent?

$W(t) = \cos(t)\left[\cos^2(t) - \sin(t)\cos(t)\right]$

$W(0) = 1$

$\to \{X_1(0), X_2(0), X_3(0)\}$ is linearly independent

$\to \{X_1(t), X_2(t), X_3(t)\}$ is linearly independent

2. (5 points) Find the general solution to the following homogeneous differential equation.

$$x'(t) = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} x(t)$$

$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\quad |A - \lambda I| = (2-\lambda)^2 - 1 = \lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 1)(\lambda - 3)$

$\lambda = 3, 1$

$\lambda = 3: \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad X_1 = X_2 \to X_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \to e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 1: \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad X_1 = -X_2 \to X_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \to e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

$X(t) = C_1 e^{3t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}$