You have 20 minutes to complete the quiz.

Variation of parameters states that a solution to $ay'' + by' + cy' = f(t)$ may be written as $y(t) = v_1(t)y_1(t) + v_2(t)y_2(t)$, where $y_1(t), y_2(t)$ are the solutions to the homogeneous equation, and

$$v_1(t) = \int \frac{-f(t)y_2(t)}{a[y_1(t)y_2'(t) - y_1'(t)y_2(t)]}\,dt \qquad v_2(t) = \int \frac{f(t)y_1(t)}{a[y_1(t)y_2'(t) - y_1'(t)y_2(t)]}\,dt.$$

1. (10 points) Find a solution to $y'' - 6y' + 9y = t^{-3}e^{3t}$

$r^2 - 6r + 9 = (r-3)^2 = 0$

$y_1 = e^{3t} \qquad y_2 = te^{3t}$

$$v_1 = \int \frac{-t^{-3}e^{3t}te^{3t}}{e^{3t}[e^{3t}+3te^{3t}] - 3e^{3t}te^{3t}}\,dt = \int -t^{-2}\,dt = t^{-1}$$

$$v_2 = \int \frac{t^{-3}e^{3t}e^{3t}}{e^{3t}e^{3t}}\,dt = \int t^{-3}\,dt = \frac{-1}{2}t^{-2}$$

$$y(t) = v_1y_1 + v_2y_2 = t^{-1}e^{3t} + \frac{-1}{2}t^{-2}te^{3t} = \boxed{\frac{1}{2}te^{3t}}$$