

You have 20 minutes to complete the quiz.

1. (10 points) All of the following questions are about the quadratic form $Q : \mathbb{R}^2 \rightarrow \mathbb{R}$ given by

$$Q(x_1, x_2) = x_1^2 + x_2^2 + 4x_1x_2$$

- Is the quadratic form positive, negative, or indeterminate?

$$Q(x) = x^T A x \quad A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \det \begin{bmatrix} 1-2 & 2 \\ 2 & -2 \end{bmatrix} = (1-2)^2 - 4$$

$$= 2^2 - 2 \cdot 2 - 4 = 0$$

$$= (2-3)(2+1) = 0$$

$$\lambda = 3, -1$$

→ indeterminate

- Find a matrix P such that $Q(x) = Q(Py)$ has no cross terms.

$$A - 3I = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$A + 1I = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} \quad P = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

~~$$Q(Py) = y^T (P^T A P) y = y^T \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} y = 3y_1^2 - y_2^2$$~~

- What is the shape of the solution set to the equation $Q(x) = 1$? (You need not graph anything).

$$Q(x) = Q(Py) = y^T (P^T A P) y = y^T \begin{bmatrix} 3 & 0 \\ 0 & -1 \end{bmatrix} y = \underline{3y_1^2 - y_2^2 = 1}$$

Hyperbola.