MATH 54 – LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS QUIZ 4A

You have 15 minutes to complete the quiz.

1. (10 points) Let $A = \begin{bmatrix} 4 & 1 \\ 3 & 6 \end{bmatrix}$

- Is 3 an eigenvalue of A? If so, compute an *eigenvector* with this eigenvalue.
- Is 4 an eigenvalue of A? If so, compute an *eigenvector* with this eigenvalue.
- Is 7 an eigenvalue of A? If so, find a basis for the corresponding *eigenspace*.

$$A - 3I = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \rightarrow \begin{matrix} x_2 \text{ free} \\ x_1 = -x_2 \end{matrix}$$

So $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is an eigenvector with eigenvalue 3.

$$A\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$$

$$\det(A - 4I) = \det\begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} = -3. \text{ So } Nul(A-4I) = \{0\}$$

4 is not an eigenvalue

$$\cancel{Nul(A-3I)}\ A - 7I = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} \sim \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}$$

$$Nul(A-7I) = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, -3x_1 + x_2 = 0 \right\}$$

$$= \left\{ x_2 \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \,\middle|\, x_2 \in \mathbb{R} \right\}$$

$$= \text{Span}\left\{ \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \right\}. \text{ So } \left\{ \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \right\} \text{ is a } \underline{\text{basis}}.$$