You have 20 minutes to complete the quiz.

1. (4 points) Write Nul A as the span of some vectors.

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 2 & 3 & -1 \\ 0 & 0 & 1 & 6 & -2 \\ 0 & 0 & 5 & 7 & -10 \end{bmatrix} \sim \left[\begin{array}{ccccc|c} 1 & -2 & 2 & 3 & -1 & 0 \\ 0 & 0 & 1 & 6 & -2 & 0 \\ 0 & 0 & 0 & -23 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

$x \in \text{Nul } A$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2x_2 - 3x_5 \\ x_2 \\ 2x_5 \\ 0 \\ x_5 \end{bmatrix} = x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

2. (4 points) Find a basis for Col A, where A is the same as above.

The pivot columns are #1, 3 & 4. So

$$\text{Col } A = \text{Span}\left\{\begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 8 \end{bmatrix}\right\} \rightarrow \text{Nul } A = \text{Span}\left\{\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}\right\}$$

And this is a basis.

3. (2 points) $B = \{b_1, b_2\} = \{5x + 1, x - 2\}$ is a basis for the vector space $\mathbb{P}_1$. Find the coordinate vector (relative to B) of $x \in \mathbb{P}_2$.

$$x = a(5x + 1) + b(x - 2)$$

$$(1)x + 0 = (5a + b)x + (a - 2b)$$

$$\rightarrow \begin{array}{l} 5a + b = 1 \\ a - 2b = 0 \end{array} \rightarrow \begin{array}{l} a = \frac{2}{11} \\ b = \frac{1}{11} \end{array}$$

$$\boxed{T_B(x) = \begin{bmatrix} \frac{2}{11} \\ \frac{1}{11} \end{bmatrix}}$$