

You have 20 minutes to complete the quiz.

1. (5 points) Put the following matrix into reduced row echelon form.

$$\begin{bmatrix} 4 & 2 & 8 & 0 \\ 3 & 3 & 6 & 0 \\ 2 & 0 & 4 & 0 \end{bmatrix}$$

$$\begin{aligned} &\sim \begin{bmatrix} 2 & 1 & 4 & 0 \\ 1 & 1 & 2 & 0 \\ 2 & 0 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 4 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 2 & 1 & 4 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \\ &\sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

2. (3 points) Find the general solution to the linear system whose augmented matrix is the following.

$$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -35 & -3 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

~~$x_5$  is free~~  
 ~~$x_4 = 4 - 9x_5$~~   
 ~~$x_3$  is free~~

$$\begin{aligned} x_5 &\text{ is free} \\ x_4 &= 4 - 9x_5 \\ x_3 &\text{ is free} \\ x_2 &= 1 + 4x_5 \\ x_1 &= 5 + 3x_5 \end{aligned}$$

3. (2 points) Write out the following problem as a linear system, a vector equation, and a matrix equation. Do not solve.

I'm visiting a farm that only has chickens and cows. If I count 16 animal legs, but only 6 animal heads, then how many chickens are there, and how many cows?

$$\begin{aligned} x_1 &= \# \text{ cows} \\ x_2 &= \# \text{ chickens} \end{aligned}$$

$$\begin{aligned} 4x_1 + 2x_2 &= 16 \\ x_1 + x_2 &= 6 \end{aligned}$$

$$\begin{bmatrix} 4 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 1 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 1 \end{bmatrix} x_2 = \begin{bmatrix} 16 \\ 6 \end{bmatrix}$$

$$\left[ \begin{array}{cc|c} 4 & 2 & 16 \\ 1 & 1 & 6 \end{array} \right]$$